STAT 5084

Grad Problem : ACF for white noise

Graduate students taking STAT 5084 should complete some "grad problems" over the course of the semester. There will be around six of these problems, and you'll need to do a good job on half of them.

The goal of this problem is to investigate the sampling distribution of autocorrelation coefficients r_k under the null hypothesis that the time series is white noise.

A white noise series $e_1, e_2, ..., e_T$ has autocorrelations r_k which are approximately i.i.d. normal with mean 0 and SD $1/\sqrt{T}$.

Check this by simulation. Randomly generate a white noise series with $T = 100$ and compute the acf. Extract the first $\ell = 20$ values of the acf as an ordinary vector. Now replicate this 1000 times to produce 1000 samples of each r_k for k from 1 to 20.

1. Make histograms for a few values of k to confirm the distribution $r_k \sim N(0, 1/\sqrt{T})$.
2. Check the correlation between a few choices of $k \neq k'$ to confirm the r_k are independent (or at least uncorrelated with each other).
3. For each of your 1000 simulated samples, compute the Ljung-Box statistic

$$Q^* = T(T+2)\sum_{k=1}^{20} \frac{r_k^2}{T-k}.$$

Plot the distribution of Q^* and confirm that it is approximately χ^2 with 20 DF.