

Graduate students taking STAT 5084 should complete some “grad problems” over the course of the semester. There will be around six of these problems, and you’ll need to do a good job on half of them.

---

The goal of this problem is to show that a  $k$  moving average “detects” linear trends. For simplicity, assume  $k$  is odd.

1. Let  $T_t = a + bt$ , where  $a$  and  $b$  are constants, and  $t = \dots, -2, -1, 0, 1, 2, 3, \dots$ . Show that the  $k$ -MA of  $T_t$  is  $T_t$ .
2. Suppose  $R_t$  are iid random variables with mean zero and variance  $\sigma^2$ . Let  $A_t$  be the  $k$ -MA of  $R_t$ . Show that the expected value  $E(A_t) = 0$  and the variance  $\text{Var}(A_t) = \sigma^2/k$ .
3. Let  $Y_t = T_t + R_t$ , and argue that the  $k$ -MA of  $Y_t$  approaches  $T_t$  for large  $k$ .