# Tic-Tac-Toe

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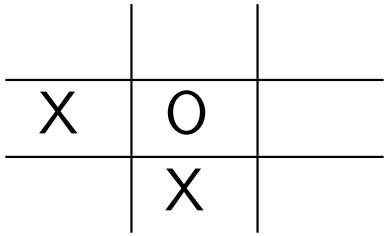
# Rules For The Simul




- I go first as X. You are O.
- Wait until I arrive at your board, then play immediately.

# The Point

Tic-tac-toe is a draw. Everyone knows this!
Use tic-tac-toe as a model problem:

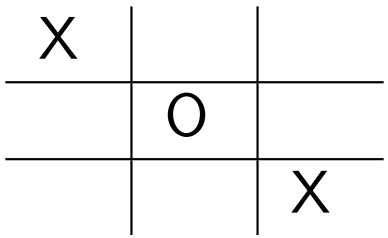
- Heuristics
- Proofs
- Classifications
- Isomorphisms

# Heuristics

1. Win (make 3-in-a-row)
2. Block (opponent's 2-in-a-row)
3. Fork (make two 2-in-a-rows)
4. Center
5. Corner

Other thoughts:

- Make a threat
- Prevent a fork
- Preserve symmetry

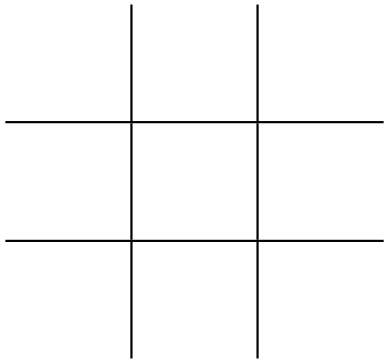


# Children

Why do people play this way?

- Stragegy as X: Play in center, then block when needed.
- Question: Playing this way, is it possible for X to lose?
- If X plays in the center, O's strategy: Play in a corner, then block.
- Question: After that, is it possible for O to lose?

# Is TTT Interesting?



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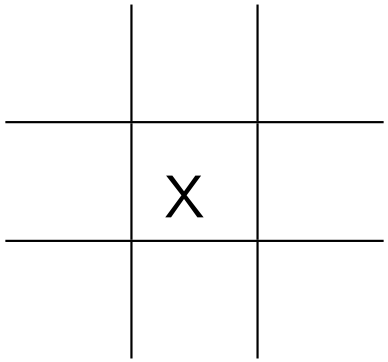

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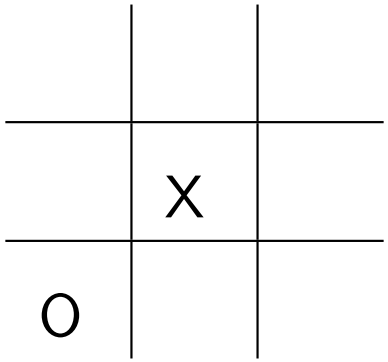

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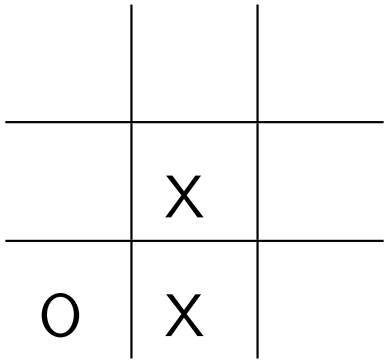

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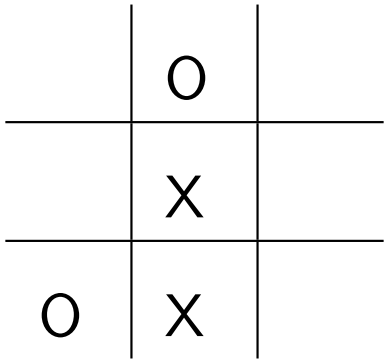

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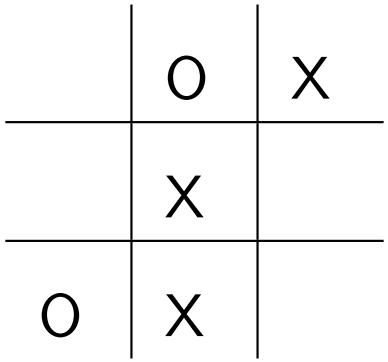
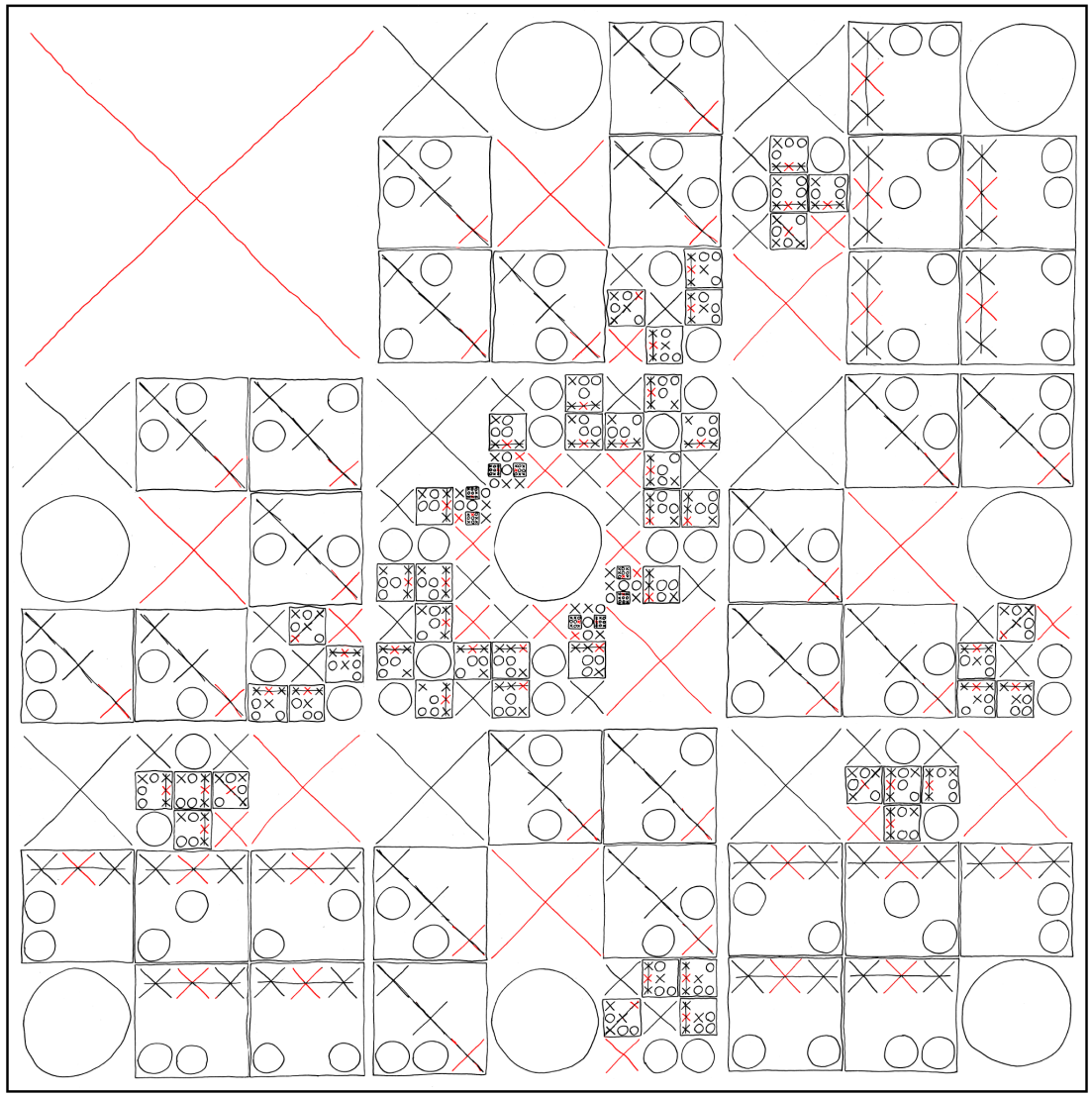



- Question: What is the most interesting possible game?
  - Maximize choices for each player, given that both play well.

# It's a Draw

# It's a Draw

Here is a proof that X cannot lose:



http://xkcd.com/832

# Classification

Classify all possible games of tic-tac-toe

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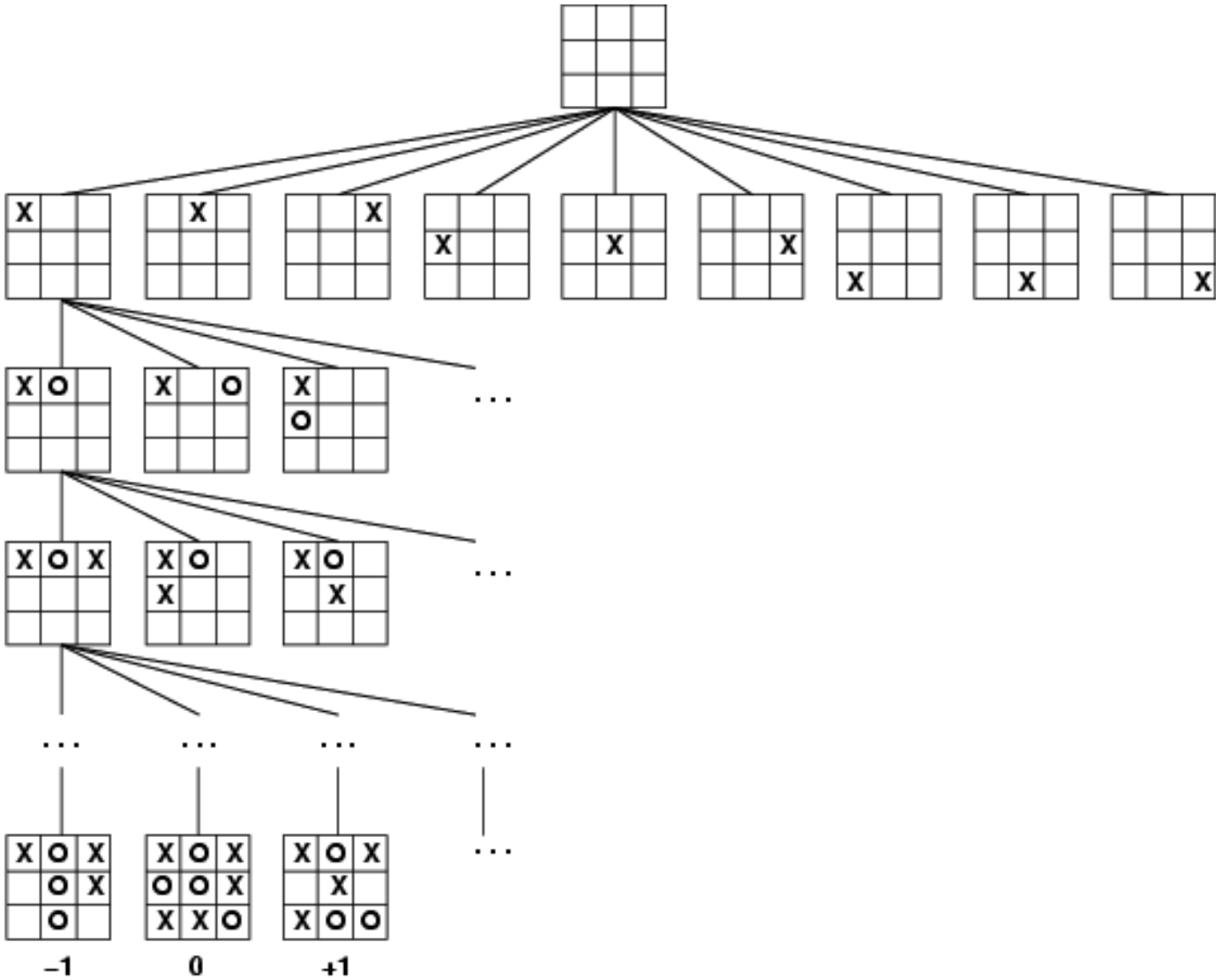

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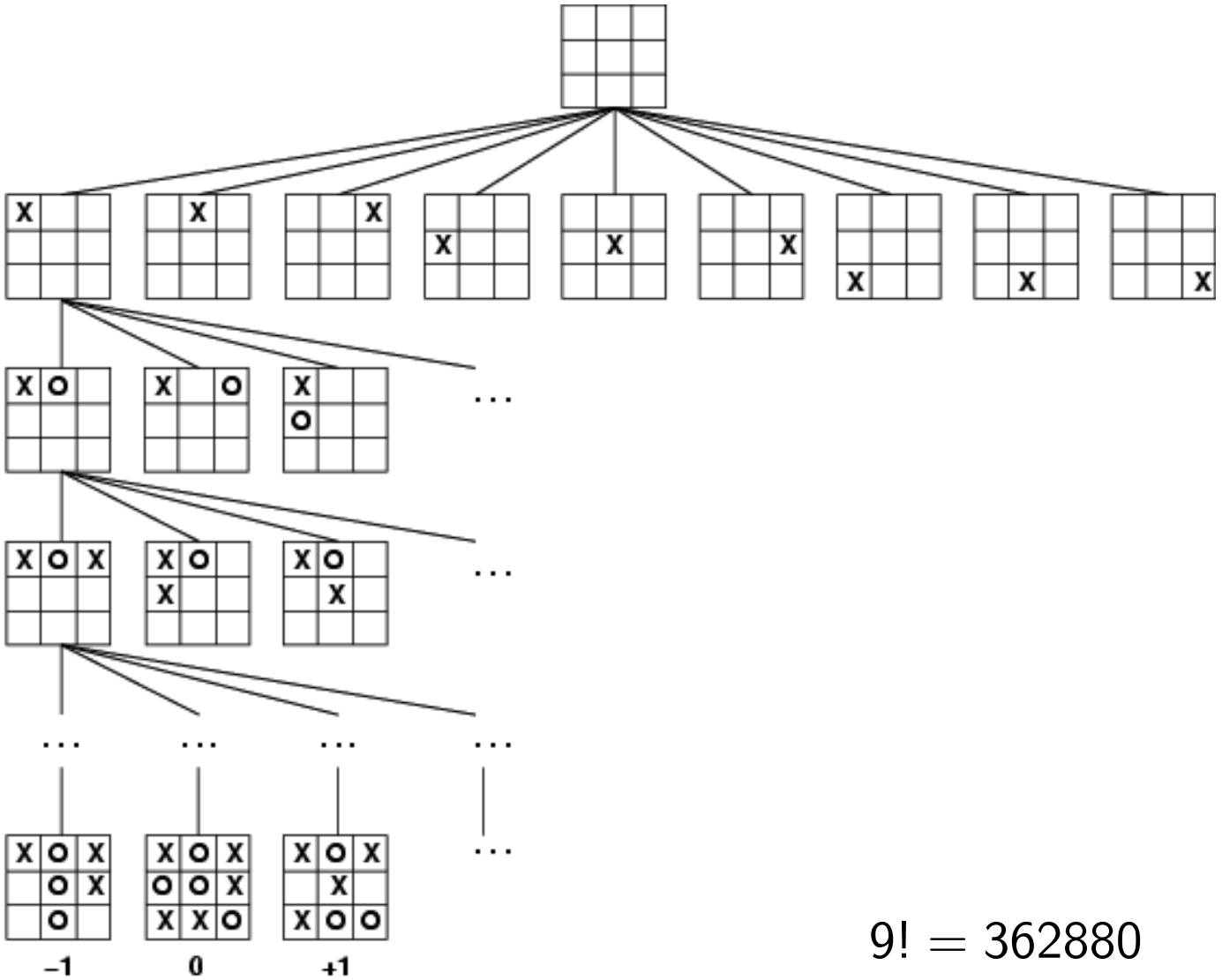
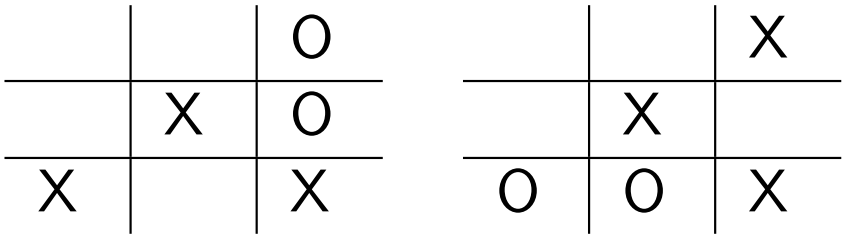



$9! = 362880$

# Isomorphism Problems

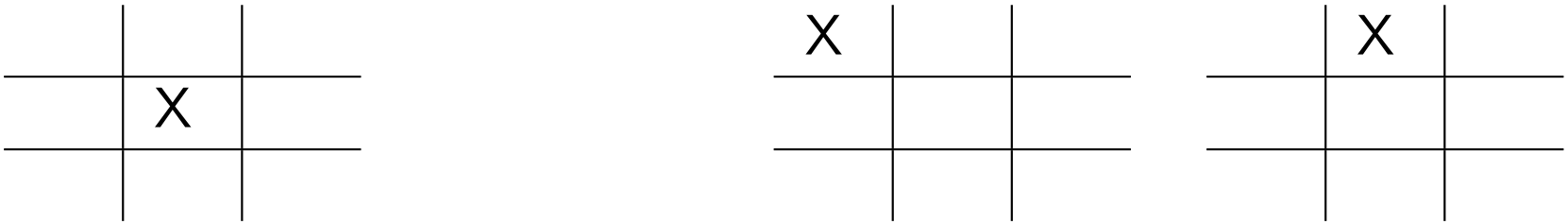
- When are two positions the same?

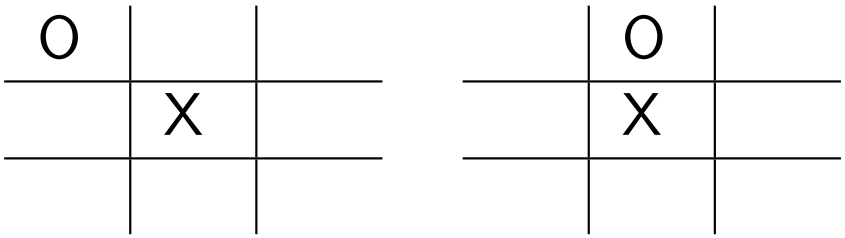


- When are two games the same?
  - What if two paths lead to the same position?

# How many games?

- Stop when game is over
- Isomorphism: Board symmetry




Steve Schaefer, Mathematical Recreations, January 2002

# Draws

Classify possible drawn positions up to isomorphism.

| | | |
|---|---|---|
| X | O | X |
| O | O | X |
| X | X | O |

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| | | |
|---|---|---|
| X | O | X |
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| | | |
|---|---|---|
| X | O | X |
| X | X | O |
| O | X | O |

- Is this all?

Yes No Exercise

# Draws

How many corners did X get?

0:

| | | |
|---|---|---|
| O | | O |
| | | |
| O | | O |

Not a draw.

1:

| | | |
|---|---|---|
| X | | O |
| | X | |
| O | | O |

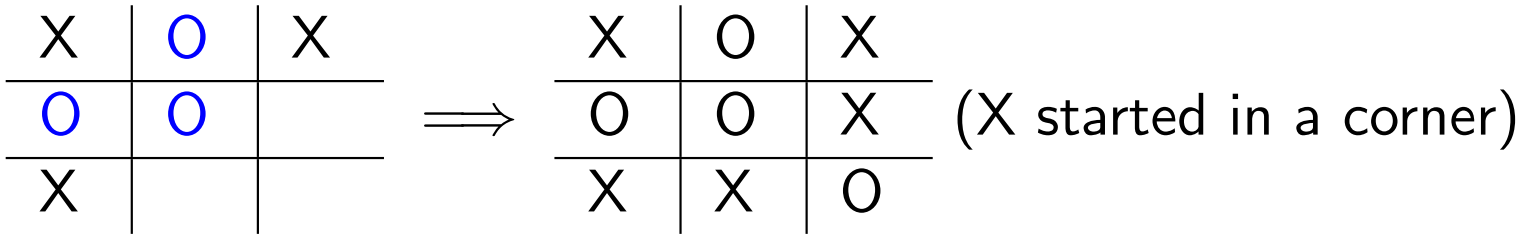
Not a draw.

4:

| | | |
|---|---|---|
| X | | X |
| | | |
| X | | X |

Not a draw.

# Draws

3:


(X started in a corner)

# Draws

3:

| | | |
|---|---|---|
| X | O | X |
| O | O | |
| X | | |

$\Longrightarrow$

| | | |
|---|---|---|
| X | O | X |
| O | O | X |
| X | X | O |

(X started in a corner)

2:

| | | |
|---|---|---|
| X | | O |
| | | |
| O | | X |

Not a draw.

# Draws

3:

| | | |
|---|---|---|
| X | O | X |
| O | O | |
| X | | |

$\Longrightarrow$

| | | |
|---|---|---|
| X | O | X |
| O | O | X |
| X | X | O |

(X started in a corner)

2:

| | | |
|---|---|---|
| X | | O |
| | | |
| O | | X |

Not a draw.

2:

| | | |
|---|---|---|
| X | O | X |
| | | |
| O | X | O |

$\Longrightarrow$

| | | |
|---|---|---|
| X | O | X |
| X | X | O |
| O | X | O |

(X started in center) and

| | | |
|---|---|---|
| X | O | X |
| X | O | X |
| O | X | O |

(Can this happen?)

# The 15 Game

- Take turns selecting numbers from 1,2,3,4,5,6,7,8,9
- No repeats: Each number can only be selected once.
- Win when exactly three of your numbers sum to 15.

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This game is isomorphic to tic-tac-toe.

# References


- Winning Ways for Your Mathematical Plays, Berlekamp, Conway, Guy.