The aperiodic monotile

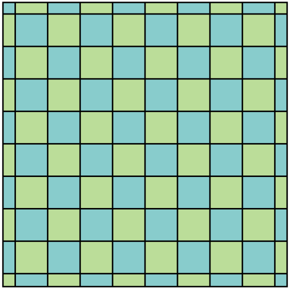
Bryan Clair

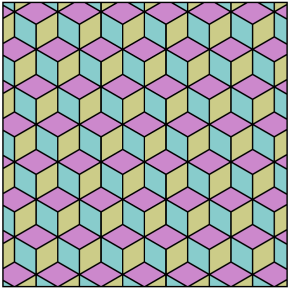
Department of Mathematics and Statistics
Saint Louis University
bryan@slu.edu

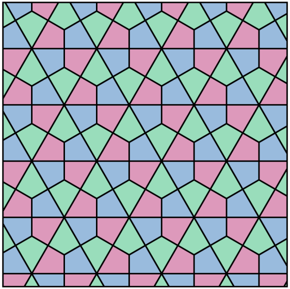
Oct 2, 2024

Tilings

What shapes can tile?







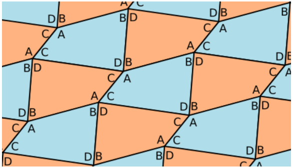
Can quadrilaterals tile?

A quadrilateral is a shape with four straight sides.

Can quadrilaterals tile?

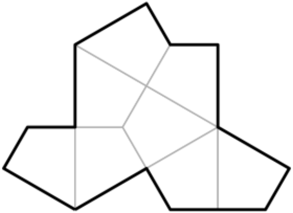
A quadrilateral is a shape with four straight sides.

Yes! Any quadrilateral can tile.



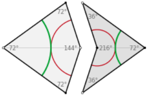
The hat monotile

Can this shape tile?



Aperiodic monotiles

The “hat” tile is the first known aperiodic monotile.
Monotile : This one shape can tile the plane.
Aperiodic : It cannot tile with a repeating pattern.

- First set of aperiodic tiles: Wang, 1961.
- Penrose set of two tiles, 1972. 

- “Hat” tile, discovered by David Smith, 2023.

Proofs?

Two key questions:

- How do we know the hat tile can actually tile the plane?
- How do we know it cannot have a repeating pattern?

Both proved in 2023 by David Smith, Joseph Samuel Myers, and Craig S. Kaplan, Chaim Goodman-Strauss.

Metatiles

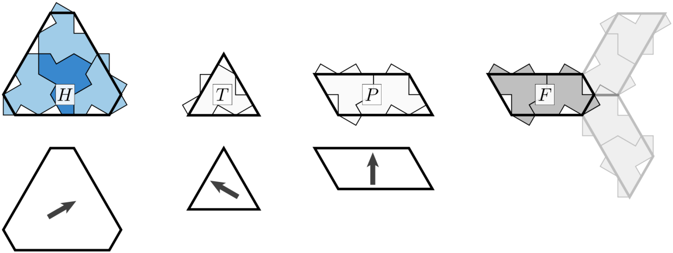
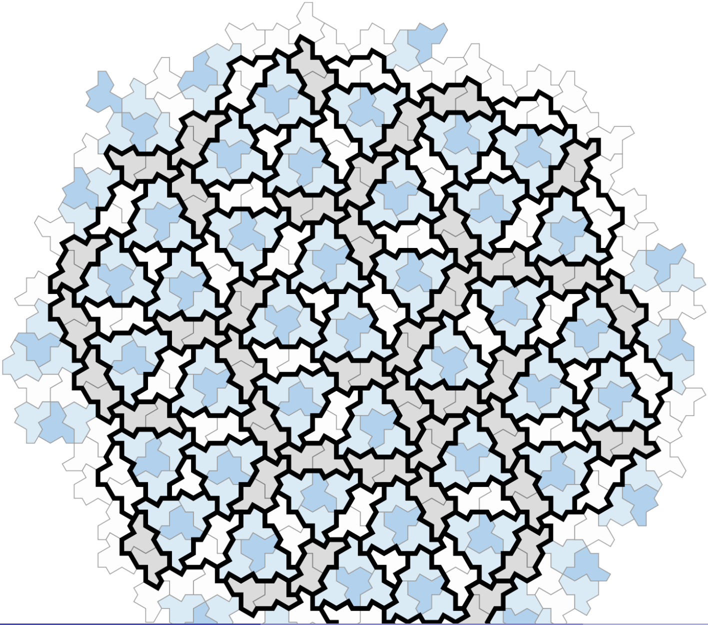
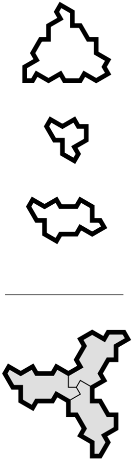


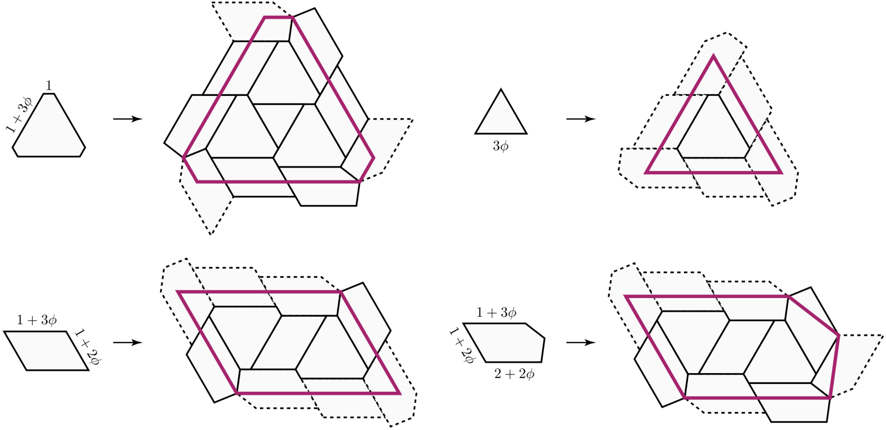
Figure 2.5: The H, T, P, and F metatiles (top), constructed by simplifying the boundaries of clusters of hats. We mark the H, T, and P metatiles with arrows when needed (bottom), to distinguish between otherwise symmetric orientations.

Metatiles in a tiling

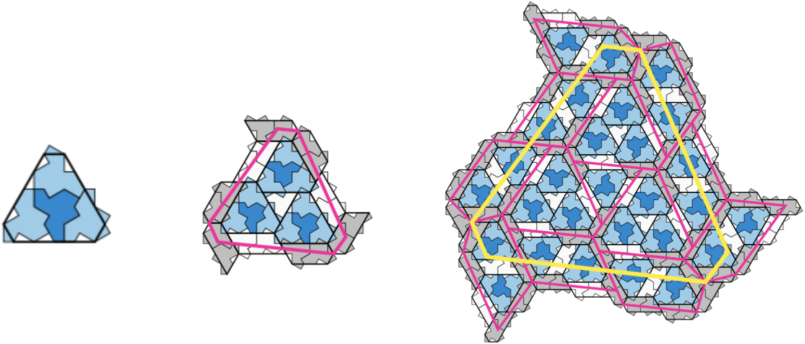




Expansion



Extension



Theorem (Tiling Extension)

If a shape can tile any size disk, then it can tile the whole plane.