Math 266 - Homework 6 - Due Friday, November 13

Modular arithmetic questions. Read Ch 3.2 and Bruce Ikegawa's notes.

1. True or false:
   (a) $32 \equiv 17 \pmod{15}$
   (b) $32 \equiv 15 \pmod{17}$
   (c) $-15 \equiv 17 \pmod{32}$
   (d) $-17 \equiv 15 \pmod{32}$
2. Show that 41 divides $2^{20} - 1$.
3. Find the remainder when $1! + 2! + 3! + \cdots + 100!$ is divided by 12.
4. Show that $111^{333} + 333^{111}$ is divisible by 7.
5. Prove that if a is an odd integer, then $a^2 \equiv 1 \pmod{8}$.
6. Prove that for any integer n, the last digit of n^4 is 0, 1, 5, or 6.
7. Find the multiplicative inverse to $22 \pmod{47}$.
8. (a) For $m \in \mathbb{N}$ and $a, b \in \mathbb{Z}$, prove that $a \equiv b \pmod{m}$ implies $a^n \equiv b^n \pmod{m}$ for all $n \in \mathbb{N}$. (Hint: use induction on n).
   (b) Give an example to show that $a^2 \equiv b^2 \pmod{m}$ does not imply $a \equiv \pm b \pmod{m}$.
9. For $a, b \in \mathbb{Z}$, and p a prime number, prove that $a^p \equiv b^p \pmod{p}$ implies $a \equiv b \pmod{p}$.
10. If $p > 2$ is prime, prove that $1^p + 2^p + 3^p + \cdots + (p-1)^p$ is divisible by p. (Hint: Fermat's Little Theorem).