Math 266 - Sample Exam 2 Questions

These are questions from class Nov. 2 that are good examples of Exam 2 type questions, though a few of them are too long or too hard to write up to make a really good exam question.

1. Prove for any sets A and B that $A \cap B = A \cup B \Leftrightarrow A = B$.
2. Find an example of a family of closed intervals with intersection $[0, 1)$.
3. List three elements in the power set of $\{0, 1\}$.
4. Use the fact that $\log ab = \log a + \log b$ for $a, b > 0$ to prove that $\log a^n = n \log a$ for $a > 0$ and $n \in \mathbb{N}$.
5. Prove
$$\prod_{i=2}^{n}(1 - \frac{1}{i}) = \frac{1}{n}$$
for all $n \in \mathbb{N}$.
6. Prove that the interior angles of an n sided polygon sum to $(n - 2)180°$.
7. Find GCD$(114, 51)$ and write it as $114x + 51y$ for some $x, y \in \mathbb{Z}$.
8. Prove that if $p|a^2 - b^2$ then either $p|a - b$ or $p|a + b$.
9. Let R be an equivalence relation on a set A. Let $S = A \times A - R$. Prove that S is symmetric.
10. Let $A = \mathbb{R} \times \mathbb{R}$. Define $(x_1, y_1) \sim (x_2, y_2)$ if $x_1 + y_1 = x_2 + y_2$.
    (a) Prove that $\sim$ is an equivalence relation.
    (b) Describe or draw the equivalence class $(0, 0)/\sim$. Describe or draw the equivalence class $(1, 1)/\sim$.