Optionally Due Monday, May 5, at the start of class. Name: _____

Math 320 – Take Home Quiz 5

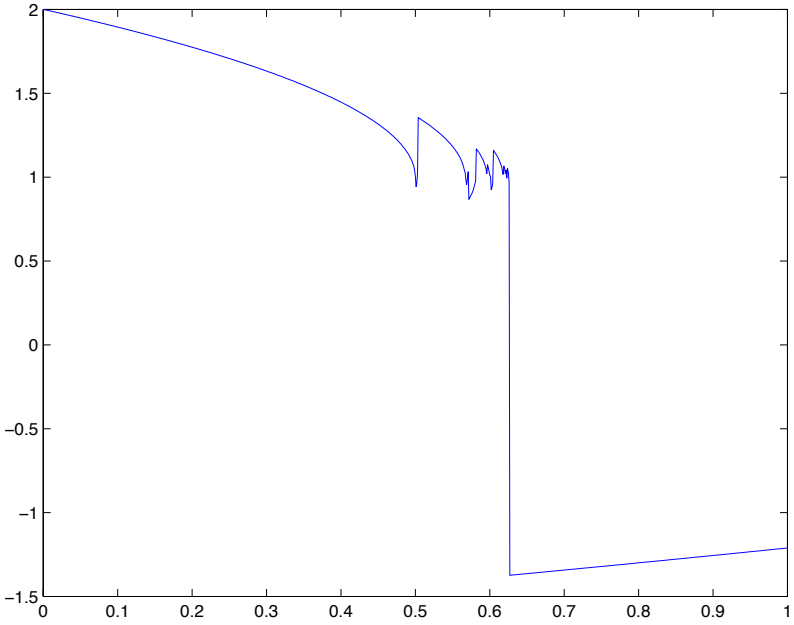
THIS QUIZ IS OPTIONAL: If you hand it in on Monday, I will grade it and it will be part of your quiz average for the semester. If you do not hand it in, your quiz average will not change.

This quiz should take you approximately 25 minutes. You may use your calculator, your book, and your notes, but do not work together and do not get help. You are allowed to use Matlab/Octave, but it is not recommended.

1. Consider the initial value problem

$$\frac{dy}{dt} = \frac{1}{1-y} \qquad y(0) = 2.$$

Solving with RK4 and $h = .001$ gives the following plot of $y(t)$:



Explain what you see, and why this happened.

2. Consider the initial value problem $\frac{dy}{dt} = t\ln(y)$, $y(0) = e$.
   Use Euler's method with a step size of $h = 0.1$ to approximate $y(0.1)$, $y(0.2)$, and $y(0.3)$.
   Give at least four decimal places in your answers.

3. (Continuing problem 2)
   (a) Use one step of the modified Euler's method to approximate $y(0.1)$.

   (b) Would you estimate that the truncation error of Euler's method for $y(0.1)$ was less than 0.1?

4. Find a Lipshitz constant L for

$$f(t,y) = \frac{e^{-3y}}{t+1}$$

on the domain $D = \{(t,y) \mid 0 \le t \le 1 \text{ and } -1 \le y \le 1\}$.

5. Suppose you use an ODE solver with a step size of $h = 0.1$ to approximate $y(5)$, where $y'(t) = y + \cos(\pi t)$ and $y(0) = 1$.

   If you cut the step size in half to $h = .05$, what effect would you expect that to have on the error at $t = 5$?

   Answer for:

   (a) Euler's method

   (b) The midpoint method

   (c) Runge-Kutta order 4