Optionally Due Monday, May 5, at the start of class. Name: ____________

Math 320 – Take Home Quiz 5

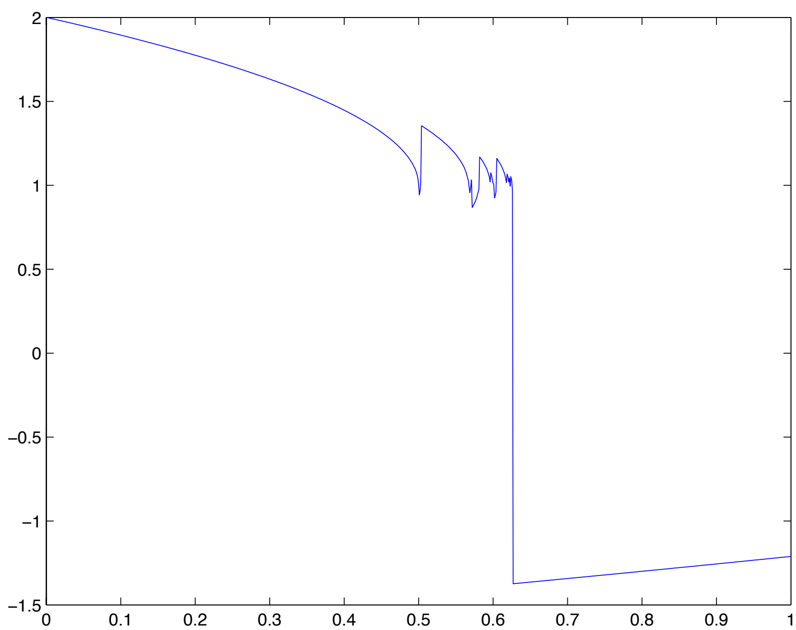
THIS QUIZ IS OPTIONAL: If you hand it in on Monday, I will grade it and it will be part of your quiz average for the semester. If you do not hand it in, your quiz average will not change.

This quiz should take you approximately 25 minutes. You may use your calculator, your book, and your notes, but do not work together and do not get help. You are allowed to use Matlab/Octave, but it is not recommended.

1. Consider the initial value problem

$$\frac{dy}{dt} = \frac{1}{1-y} \qquad y(0) = 2.$$

Solving with RK4 and $h = .001$ gives the following plot of $y(t)$:



Explain what you see, and why this happened.

Solution: $f(t, y) = 1/(1 - y)$ is not Lipshitz for any domain containing $y = 1$, and solutions to this differential equation fail to exist after some finite time.

The solution is fine until around $t = 0.5$, when $y \approx 1$. At this point, the slope f is very large and sends the approximate solution far from $y = 1$. The solver then returns towards $y = 1$ and repeats the process.

2. Consider the initial value problem $\frac{dy}{dt} = t\ln(y)$, $y(0) = e$.

   Use Euler's method with a step size of $h = 0.1$ to approximate $y(0.1)$, $y(0.2)$, and $y(0.3)$.

   Give at least four decimal places in your answers.

   Solution:
   $$y(0.1) \approx 2.7183; \quad y(0.2) \approx 2.7283; \quad y(0.3) \approx 2.7484$$

3. (Continuing problem 2)

   (a) Use one step of the modified Euler's method to approximate $y(0.1)$.

   Solution: $y(0.1) \approx 2.7233$

   (b) Would you estimate that the truncation error of Euler's method for $y(0.1)$ was less than 0.1?

   Solution: The difference between Euler's approximation and the modified Euler approximation is 0.005. Dividing by $h = 0.1$ gives a reasonable estimate of 0.05 for the truncation error in the Euler approximation, which is certainly less than 0.1.

4. Find a Lipshitz constant L for

$$f(t, y) = \frac{e^{-3y}}{t+1}$$

on the domain $D = \{(t, y) \mid 0 \le t \le 1 \text{ and } -1 \le y \le 1\}$.

Solution:

$$\left|\frac{\partial f}{\partial y}\right| = \left|\frac{-3e^{-3y}}{t+1}\right| \le \frac{3e^3}{0+1}$$

so the best possible Lipshitz constant is $L = 3e^3 \approx 60.3$.

5. Suppose you use an ODE solver with a step size of $h = 0.1$ to approximate $y(5)$, where $y'(t) = y + \cos(\pi t)$ and $y(0) = 1$.

If you cut the step size in half to $h = .05$, what effect would you expect that to have on the error at $t = 5$?

Answer for:

(a) Euler's method

(b) The midpoint method

(c) Runge-Kutta order 4

Solution: a. Should cut error in half. b. Should cut error by 1/4. c. Should cut error by 1/16.

I recommend trying this out for yourself. The actual error ratios I got are 2.006 for Euler's, 3.997 for midpoint, and 15.044 for RK4.