This quiz is for practice only, not graded. Name: ____________________

# Math 320 – Take Home Quiz 2

This quiz should take you approximately 25 minutes. You may use your calculator, your book, and your notes, but do not work together and do not get help. You are allowed to use Matlab/Octave, but it is not recommended.

(10) 1. Show $10^{-n^2}$ converges linearly to 0.

> **Solution:**
> $$\lim_{n\to\infty} \frac{10^{-(n+1)^2}}{10^{-n^2}} = \lim_{n\to\infty} 10^{n^2-(n+1)^2} = \lim_{n\to\infty} 10^{-2n-1} = 0.$$
> Since this limit is less than 1, the sequence converges linearly to 0.

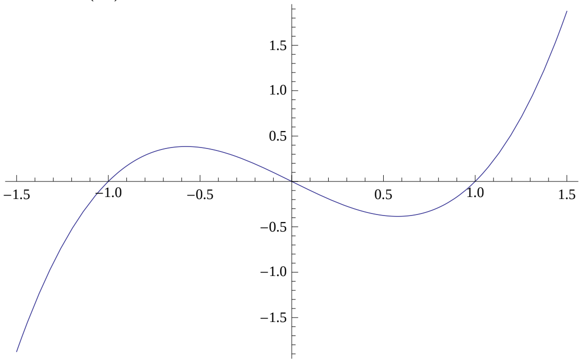
(10) 2. Give an example of a non-constant polynomial $g(x)$ so that iterating $x_0 = 1.7, x_1 = g(x_0), x_2 = g(x_1), \ldots$ converges to $\sqrt[3]{5}$.

> **Solution:** We're looking for a root of $f(x) = x^3 - 5$. With Newton's method, $g(x) = x - \frac{x^3-5}{3x^2}$. Another approach is to put $g(x) = x + cf(x)$ for some $c$ which makes $g'(\sqrt[3]{5}) < 1$. For example $g(x) = x - 0.1(x^3 - 5)$ works.

(10) 3. The function $f(x) = x^2 - x - 1$ has roots $\varphi = \frac{1+\sqrt{5}}{2}$ and $\bar{\varphi} = \frac{1-\sqrt{5}}{2}$. The larger root $\varphi \approx 1.618$ is known as the *golden mean*. Use Newtons's method with $x_0 = 2$ to compute $x_1$ and $x_2$. However, do not use decimal approximations – carry out your computations exactly so that you get fractions which are good approximations to $\varphi$.

> **Solution:**
> $$g(x) = x - \frac{f(x)}{f'(x)} = \frac{x^2+1}{2x-1}.$$
> $$g(2) = 5/3 \quad g(5/3) = 34/21$$
> Note that these will always be Fibonacci numbers: $1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots$.

(10) 4. Let $f(x) = x^3 - x$, as shown below. Apply Newton's method to solve $f(x) = 0$.




(a) Begin with initial guess $x_0 = 0.3$. Which root does Newton's method converge to?

(b) Begin with initial guess $x_0 = 0.8$. Which root does Newton's method converge to?

(c) Begin with initial guess $x_0 = 0.5$. Which root does Newton's method converge to?

(d) Begin with initial guess $x_0 = 0.455$.Which root does Newton's method converge to?

**Solution:** a. 0; b. 1; c. -1; d. 1

(10) 5. Behold a function $f(x)$ graphed below, along with the line $y = x$. Draw an accurate cobweb plot starting at $x_0 = 0.2$, to see what will happen if you iterate this function.