Due Wednesday February 5, in class Name: _____

Math 320 – Take Home Quiz 1

This quiz should take you approximately 25 minutes. You may use your calculator, your book, and your notes, but do not work together and do not get help. You are allowed to use Matlab/Octave, but it is not recommended.

(10) 1. (a) Give an example of a number p and an approximation p^* so that the absolute error is bigger than 1 but the relative error is less than .001

(b) Give an example of a number p and an approximation p^* so that the absolute error is less than .001 but the relative error is larger than 1.

(10) 2. Compute $2^{-1} + 2^{-52}$ to 20 significant digits.

(10) 3. The function $f(x) = 3x - e^x$ has a root in the interval $[1, 2]$. Show the first three steps of the bisection method to find this root. How many steps will it take to find the root to within .001?

(10) 4. Consider the series

$$\frac{\pi}{4} = \frac{3}{4} + \frac{1}{2 \times 3 \times 4} - \frac{1}{4 \times 5 \times 6} + \frac{1}{6 \times 7 \times 8} - \frac{1}{8 \times 9 \times 10} + \cdots$$

Approximately how many terms of this series do you need in order to compute $\frac{\pi}{4}$ to 150 decimal places?

(10) 5. Show that $g(x) = 0.5\cos(x)$ has a fixed point $p \in [0, 1]$, and that fixed point iteration $x_n = g(x_{n-1})$ converges to p for any $x_0 \in [0, 1]$.

Give a bound for the absolute error in the approximation $x_{20} \approx p$ after 20 steps of fixed point iteration. Your answer should be valid for any $x_0 \in [0, 1]$.