Read BF pages 276, chapter 5.4, chapter 5.5, pages 339-341, pages 348-349.

Exercises

Chapter 5.4 # 3a, 15a, 23a, 28

Problem A : Show that when $f(t, y) = f(t)$ does not depend on y, then order 4 Runge-Kutta is the same as Simpson's rule.

MATLAB/Octave

1. Implement the Runge-Kutta order 4 method for solving ODE's. Write a MATLAB function

```
function [t,y] = rungekutta4( f, tspan, y0, n)
     %  Apply the RK4 method to solve y' = f(y,t), y(a) = y0,
     %  on the interval tspan(1) <= t <= tspan(2) with n steps.
```

   Then use it to solve the ODE $y' = ty^2$, $0 \le t \le 1$, $y(0) = 1$. Do this for $n = 10, 100, 1000$ steps. For each choice of n, find the error between your computed value at $t = 1$ and the correct value at $t = 1$ (which is $y(1) = 2$). Make a table of these errors, and observe how they change when n is multiplied by 10.

2. Consider the stiff ODE $y' = 100 - y$, $0 \le t \le 200$, $y_0 = 5$. Solve this using `ode45`, `ode23s`, and `rungekutta4` (with $N = 1000$). In each case,

   - Plot the results for $t \in [100, 200]$ and $y \in [99.8, 100, 2]$. Describe the result (no need to print).
   - Use `tic` and `toc` to compute the time it took for the computations.
   - How many points did each method use?
   - Compare the various methods for suitability to this ODE.