Read BF Chapter 4.6, 5.1, 5.2, pg 286.

Exercises

Chapter 4.6 # 1a, 6a

Chapter 5.1 # 3a, 4abd

Chapter 5.2 # 1bc, 3bc, 12 (see problem C)

Problem A : Suppose you want to compute $\int_0^{2\pi} e^{-x}\cos(x)dx$ to within 10^{-6}. Use the error term for composite Simpson's rule to find the number of function evaluations required for this level of accuracy.

Now use adaptive quadrature (the `quadgui` function) to do the computation. How many function evaluations did `quadgui` require?

Problem B : For each part, find a Lipshitz constant L for the y variable for the function $f(t, y)$ on the domain $D = \{(t, y) \mid 0 \le t \le 1 \text{ and } -1 \le y \le 1\}$.

a. $f(t, y) = e^{t-y}$ b. $f(t, y) = \frac{y^2}{1+t}$ c. $f(t, y) = |\cos(ty)|$

Problem C : What happens in Ch. 5.2 problem 12 if $h = .2$?

MATLAB/Octave

1. Consider the initial value problem $y' = y\sin(t)$, $0 \le t \le 20$, $y(0) = 0.1$.
   (a) Use `dirfield.m` to plot a slope field for this ODE, and then use MATLAB's `ode45` to compute and plot a solution on the same plot. How many steps did `ode45` use to compute the solution?
   (b) Use `euler.m` and `eulermod.m` (from class) to compute and plot solutions on the same figure as part (a). Use $n = 100$ steps.

   Print your plot with the slope field and three solutions. What do you observe?

2. Implement the midpoint method for solving ODE's (pg 286). Write a MATLAB function

```
function [t,y] = midpoint( f, tspan, y0, n)
     %  Apply the midpoint method to solve y' = f(y,t), y(a) = y0,
     %  on the interval tspan(1) <= t <= tspan(2) with n steps.
```

   Then use it to solve the ODE $y' = ty^2$, $0 \le t \le 1$, $y(0) = 1$. Do this for $n = 10, 100, 1000, 10000,$ 100000 steps. For each choice of n, find the error between your computed value at $t = 1$ and the correct value at $t = 1$ (which is $y(1) = 2$). Make a table of these errors, and observe how they change when n is multiplied by 10.