Read BF Chapter 1.2 on Nested Arithmetic, Chapter 2.6, Chapter 3.1

Exercises

Chapter 2.6 # 12 (you can use matlab question 1 to help)

Chapter 3.1 # 1a, 5a*, 7a*, 14, 17
* You might want to do more of 5,6,7,8 if you need practice.

Problem A: Suppose you approximate $f(x) = x^3$ with a degree two Lagrange interpolating polynomial using nodes 0, 1, and a. Which choice of a gives the best approximation overall? Here, use the maximum error on the interval [0,1] as the measure of "best".

MATLAB/Octave

1. Write a function `horner(a,x)` that takes a vector $a = [a_n a_{n-1} \dots a_0]$ and evaluates $p(x) = a_n x^n + \cdots + a_2 x^2 + a_1 x + a_0$ using Horner's method. (Note that matlab actually has a built-in function `polyval` which does this.)

   Demonstrate your function by computing the value of the 5th Bessel polynomial at .01:

   `horner([945,945,420,105,15,1],.01)`

2. (Runge's example)

   The goal of this problem is to investigate interpolating polynomials for the function

   $$f(x) = \frac{1}{1+x^2}$$

   on the interval $[-5, 5]$.

   You should make one plot, with vertical range from -1 to 2, which shows all of the following:

   - The function $f(x)$
   - The interpolating polynomial for f with nodes at $-5, -4, -3, \dots, 3, 4, 5$.
   - The interpolating polynomial for f with nodes at $-5, -4.5, -4, \dots, 3.5, 4, 4.5, 5$.
   - The interpolating polynomial for f with nodes at $-5, -4.9, -4.8, \dots, 4.7, 4.8, 4.9, 5$

   You'll want to use the `polyinterp.m` function provided in class for this.

   `ylim([-1 2])` will set the appropriate y-axis for your plot.

   Question: Estimate for which x these polynomials give a good approximation to f.