

Read BF Chapter 1

Exercises

Chapter 1.2 # 1a, 1h, 9a, 15

Chapter 1.3 # 3,4

Problem A: The Taylor series for $\log(1+x)$ at $x=0$ is given by

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

Show that $\log(2) = 2\log(\frac{12}{10}) - \log(\frac{8}{10}) - \log(\frac{9}{10})$. Use the series to compute $\log(\frac{12}{10})$, $\log(\frac{9}{10})$, and $\log(\frac{8}{10})$ to at least five decimal places, and then compute $\log(2)$.

You can use a calculator or computer if you must, but to really see the beauty of this approach you should do it totally by hand. Isaac Newton used this method when computing tables of logarithms. (Although this is not “Newton’s Method”).

MATLAB/Octave

1. Write a function `biroot(x)` that computes the square root of `x` using the bisection method. Start your interval with endpoints $a=1$ and $b=x$. You can assume $x > 1$.
2. Let

$$\gamma_n = 1 + 1/2 + 1/3 + 1/4 + \cdots + 1/n - \log(n).$$

Euler’s gamma constant is

$$\gamma = \lim_{n \rightarrow \infty} \gamma_n \approx 0.5772156649,$$

and it is not known if γ is a rational number.

Write a function `eulergamma(n)` that computes γ_n .

Make a table showing how large n needs to be before γ_n agrees with γ to 1,2,3,4,5, and 6 decimal places.