Monday, May 5

Math 320 – Review Questions

1. Given an ODE problem $y' = f(t, y)$, $a \leq t \leq b$, $y(a) = y_0$, let $h = \frac{b-a}{N}$ and $t_i = a + ih$. The Backwards Euler Method for solving puts $w_0 = y_0$ and computes:

$$w_{i+1} = w_i + hf(t_{i+1}, w_{i+1}).$$

   This method is implicit - the variable w_{i+1} appears on both sides of the equation, and you need to solve for w_{i+1} to take the step.

   Use this method with $N = 5$ to solve $y' = -10y$, $0 \leq t \leq 1$, $y_0 = 1$, and compute the approximation to $y(1)$.

   Repeat with $N = 10$.

2. Consider the differential equation $y' = ty^\alpha$, where $0 \leq t \leq 1$, $y_0 = 1$, and $\alpha > 0$ is a constant.

   (a) Find a Lipshitz constant (depending on α) for f in the y variable on the domain

$$D = \{(t, y) | 0 \leq t \leq 1 \quad \text{and} \quad 1 \leq y \leq 2\}.$$

   (b) If you use Eulers method with $n = 100$, will you get a more accurate solution for the $\alpha = 1$ problem or the $\alpha = 2$ problem?

3. Computing the integral $\int_0^1 e^{-x^2} dx$ with Simpson's rule and $h = 0.25$ gives 0.746855. Using $h = 0.125$ gives 0.746826.

   Use extrapolation to compute a more accurate result.

4. Show that the midpoint rule for quadrature has degree of precision equal to one.

5. (a) Perform one step of Euler's method on the differential equation $y' = y + t$ with $y_0 = 1$, $0 \leq t \leq 0.5$ and $h = 0.5$.

   (b) Repeat with the modified Euler's method.

   (c) Use part (b) to estimate the error in your result from part (a).