Thursday, April 3

Math 320 – Review Questions

(10) 1. The polynomial $\chi(t) = t^7 - 12t^6 + 67t^5 - 230t^4 + 529t^3 - 814t^2 + 775t - 352$ is important in graph theory.

(a) Write this polynomial in nested form.

Solution: $-352 + t(775 + t(-814 + t(529 + t(-230 + t(67 + t(-12 + t))))))$

(b) How many multiplications are needed to compute $\chi(t)$ using Horner's algorithm?

Solution: Six.

(10) 2. The "logarithmic integral" function li is defined by $\mathrm{li}(x) = \int_0^x \frac{dt}{\ln t}$.

(a) Given that $\mathrm{li}(4) = 2.96759$ and $\mathrm{li}(5) = 3.63459$, use linear interpolation to compute an approximation to $\mathrm{li}(4.3)$.

Solution: Linear interpolation gives $\mathrm{li}(x) \approx 2.96759\frac{x-5}{4-5} + 3.63459\frac{x-4}{5-4}$ so $\mathrm{li}(4.3) \approx 3.16769$.

(b) Give a good bound on the error in your approximation from part (a).
(Hint: By the fundamental theorem of calculus, $\frac{d}{dx}\mathrm{li}(x) = \frac{1}{\ln x}$)

Solution: First $\mathrm{li}''(x) = \frac{-1}{x\ln^2(x)}$ is maximized on the interval $[4,5]$ at $x = 4$, where $\mathrm{li}''(4) \approx 0.1301$. Now the error bound for degree 1 interpolation is

$$\left|\frac{\mathrm{li}''(\xi)}{2!}(4.3-4)(4.3-5)\right| \leq \frac{0.1301}{2}(.3)(.7) = 0.01366$$

Actually, $\mathrm{li}(4.3) = 3.17847$ so the error in part (a) was about 0.011, reasonably close to the bound.

(10) 3. Find a cubic polynomial p that takes all of the following values:

x	0	1	2	3	4	5
$p(x)$	-82	-19	0	-1	2	33

Solution: Constructing divided differences gives:

```
-82       -19        0        -1        2        33
      63        19       -1         3        31
          -22       -10        2        14
                4         4         4
                     0         0
                          0
```

so that $p(x) = -82 + 63x - 22x(x-1) + 4x(x-1)(x-2)$.

(10) 4. A smooth function f has values

x	-.2	-.1	0	.1	.2
$f(x)$	0.67474	0.73282	0.78540	0.83298	0.87606

Compute $f'(0)$ as well as you can. Explain your method.

Solution: This function is $f(x) = \text{atan}(x+1)$ and $f'(0) = .5$ exactly.

From worst to best:

- Forward difference with $h = .2$ gives $\frac{f(.2)-f(0)}{.2} = .4533$, error of 4.7×10^{-2}.
- Forward difference with $h = .1$ gives $\frac{f(.1)-f(0)}{.1} = .4758$, error of 2.4×10^{-2}.
- Central difference with $h = .2$ gives $\frac{f(.2)-f(-.2)}{.4} = .5033$, error of 3.3×10^{-3}.
- Richardson extrapolation on the forward differences gives

  $$.4533 + (.4533 - .4758) = .4983,$$

  with an error of 1.7×10^{-3}.
- Central difference with $h = .1$ gives $\frac{f(.1)-f(-.1)}{.2} = .5008$, error of 8×10^{-4}.
- Richardson extrapolation on the central differences gives

  $$.5008 + \frac{(.5008 - .5033)}{3} = .49997,$$

  with an error of 3×10^{-5}.

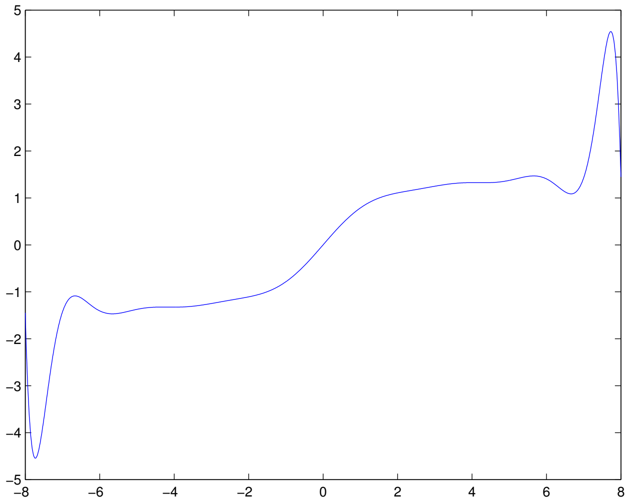
(10) 5. We used a MATLAB function `polyinterp(x,y,u)` to compute $P(u)$ where P is the interpolating polynomial with $P(x_i) = y_i$.

(a) Write a MATLAB command (or commands) make a plot of P on the interval $[-8, 8]$, where P interpolates $\text{atan}(x)$ for $x = -8, -7, -6, -5, \ldots, 5, 6, 7, 8$.

Solution:

```
>> x=[-8:8];
>> y=atan(x);
>> u=[-8:.01:8];
>> plot(u,polyinterp(x,y,u))
```

(b) The result of part (a) would look like this:



Explain the picture.

Solution: Between -4 and 4, the picture looks pretty well like the arctangent function. However, this high-degree interpolating polynomial starts to oscillate near the ends of the region, and does not approximate the function well at all near $x = \pm 8$.