SLU MISSOURI COLLEGIATE MATH TEAM – QUALIFYING EXAM 2018
MARCH 26, 2018

Instructions:

- Please work on these problems independently.

- Explain your ideas as clearly as possible.

- Don't worry about solving everything, just do what you can.

- Please submit your work electronically (photo/scan/typed) by 5pm on Friday, March 30th.

THE PROBLEMS:

1. For $x > 0$ let R_x be the rectangle with its lower-left corner at the origin and its upper-right corner at the point (x, e^{-x}). What is the maximum possible area of R_x, $x > 0$?

2. Compute $\displaystyle\int_0^1 \frac{1}{1 + x^{\frac{1}{3}}}\, dx$.

3. A *bitstring* of length n (n a positive integer) is a sequence of n ones and zeros, e.g., 0010 is a bitstring of length 4.

   a. How many bitstrings of length 10 contain three or more 1's?

   b. How many bitstrings of length 10 contain at least three consecutive 1's?

4. A vector $x = (x_1, x_2, \ldots, x_n)$ is said to be *s-sparse* if at most s of the coordinates x_j $(1 \leq j \leq n)$ are nonzero. Solve the following system of linear equations given the fact that $x = (x_1, \ldots, x_5)$ is 2-sparse.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 4 \\ -1 & 1 & 1 & -2 & -1 \\ -1 & -2 & 1 & 0 & -2 \\ 0 & -1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \\ -2 \\ -3 \end{bmatrix}$$

5. *Texas Hold'em* is a form of Poker in which each player is dealt two cards and all players share community cards that are dealt to the table. After the players receive their two cards, the community cards are dealt in three steps. First, the dealer places three cards on the table, which are referred to as the *flop*. Second, the dealer places a fourth card on the table, known as the *turn*. Finally, the dealer places a fifth card on the table, which is called the *river*. (Texas Hold'em uses a standard 52-card deck.)

   Suppose that a player receives the Four of Clubs and the Five of Diamonds at the start of the game. Using the community cards, this player hopes to make a *straight*, which is a hand consisting of five consecutive cards ranging from A-2-3-4-5 up to 10-J-Q-K-A.

   a. What is the probability that the player can form a straight after the flop has been dealt?

   b. What is the probability that the player can form a straight (using the original two cards) after all five community cards have been dealt?