

MARCH 25, 2017

EXPLAIN YOUR WORK CAREFULLY TO MAXIMIZE CREDIT.

PROBLEMS:

1. The number 99 was multiplied by an integer k to obtain an integer of seven decimal digits, but two of the digits got blotted out on the paper. The product was $62ab427$, but the digits a and b are illegible. Determine all possible values of a and b .

Source: Iowa Collegiate Mathematics Competition, 2017.

2. Find the sum of the series

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{9} + \frac{1}{12} + \cdots,$$

where the terms are the reciprocals of the positive integers whose only prime factors are twos and threes.

Source: Missouri Collegiate Mathematics Competition, 2014.

3. Notice that 73 can be written as a sum of two consecutive positive integers, $73 = 36 + 37$. Prove that no longer sum of consecutive positive integers equals 73.

4. Suppose that the function f satisfies $f'(x) = 1 + f(x)$ for all x . If $f(2) = 3$ find:

- (a) $f^{(10)}(2)$ where $f^{(10)}$ denotes the 10th derivative of f ;
- (b) $f(3)$.

Source: Iowa Collegiate Mathematics Competition, 2017.

5. For real $a > 0$ define the sequence $\{x_n\}$ by

$$x_{n+1} = a(x_n^2 + 4), \quad x_0 = 0.$$

Determine necessary and sufficient conditions on a for $\lim_{n \rightarrow \infty} x_n$ to exist and be finite.

Source: Missouri Collegiate Mathematics Competition, 2012.