

SLU MISSOURI COLLEGIATE MATH TEAM – QUALIFYING EXAM 2016

APRIL 3, 2016

EXPLAIN YOUR WORK CAREFULLY.

PROBLEMS:

1. (Iowa Collegiate Math Competition, 2015) We are given six jugs, the first five containing 2 liters of water each, and the sixth containing one liter. At each step we can select any two jugs, and then pour water from one into another until they contain equal amounts of water. Is it possible to make the quantities of water in all jugs equal? Explain your answer.
2. A bag contains 10 balls, numbered 0-9. Three balls are drawn:  $b_1$ ,  $b_2$ , and  $b_3$ . What is the probability that  $b_1 < b_2 < b_3$ ?
3. Compute  $\int_0^{\infty} \frac{\ln x}{x^2 + 2x + 4} dx$ .
4. (Iowa Collegiate Math Competition, 2015) Find all right triangles whose sides are positive integers and whose perimeter is numerically equal to its area.
5. Let  $N$  be the random variable representing the number of coin flips required for two consecutive flips to have the same result. Assuming the coin is fair, what is the expected value of  $N$ ?