SLU Math Team 2013 Qualifying Problems

Return your work to Dr. Clair's office (Ritter 110) before 4pm on Tuesday, April 2. Even if you feel you got none of the problems, you need to hand in something (a blank sheet of paper with your name on it?) to declare your desire to go on the trip.

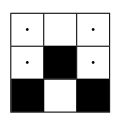
1. In the product $1! \cdot 2! \cdot 3! \cdot \cdots \cdot 99! \cdot 100!$, prove that erasing one of the factors of the product results in a perfect square.

2. Two positive numbers real numbers are given. Their sum is less than their product. Prove their sum is at least 4.

3. Let $f : [1, 2] \to \mathbb{R}$ be a continuous function such that $\displaystyle\int_1^2 f(x)dx = 0$.

   Prove there is a number $c \in (1, 2)$ such that $cf(c) = \displaystyle\int_c^2 f(x)dx$.

4. Imagine a board made out of squares, with each square colored either black or white. Say that a board is *wrecked* if no rectangular sub-board has all four of its corners the same color. For example, the board below is not wrecked because the dots form a rectangle with four white corners:

   For which integers $n > 1$ is there an $n \times n$ square board which is wrecked?

5. Is the following series convergent or divergent?

$$\frac{1}{2} \cdot \frac{19}{7} + \frac{2!}{3^2} \cdot \left(\frac{19}{7}\right)^2 + \frac{3!}{4^3} \cdot \left(\frac{19}{7}\right)^3 + \frac{4!}{5^4} \cdot \left(\frac{19}{7}\right)^4 + \cdots$$