SLU Math Team 2011 Qualifying Problems

Return your work to Dr. Clair's office (Ritter 110) before 3pm on Tuesday, March 22. Even if you feel you got none of the problems, you need to hand in something (a blank sheet of paper with your name on it?) to declare your desire to go on the trip.

1. Let N be the product of all positive integers which divide 1,000,000,000. Find $\log_{10}(N)$.

2. Evaluate
$$\int_0^{\sqrt{2\pi}} \int_{y/2}^{\sqrt{\pi/2}} \sin(x^2) dx dy.$$

- 3. Find a real valued function f so that $f(f(x)) = -\frac{1}{x}$ for all positive x.
- 4. Consider the hexagon shown below with 120° angles, short sides of length 3, and long sides of length 5. Can it be tiled (covered without overlaps) by rhombuses with side length 1 and angles $60^{\circ} 120^{\circ} 60^{\circ} 120^{\circ}$?



5. Take two rays from O making an angle $0 < \theta < \pi$. Let C_1 be the circle which is tangent to both rays and which has center at distance 1 from O. Let C_2 be the circle which is tangent to both rays and to C_1 , and which is smaller than C_1 . Find θ so that C_2 is a large as possible.