

SLU Math Team 2010 Qualifying Problems

Return your work to Dr. Clair on or before Thursday, March 25.

Even if you feel you got none of the problems, you need to hand in something (a blank sheet of paper with your name on it?) if you want to go on the trip.

1. Calculate $\lim_{x \rightarrow 1} \frac{\int_x^{x^2} e^{-t^2} dt}{x - 1}$.
2. Suppose that $r > 0$ is a rational approximation to $\sqrt{5}$. Prove that $\frac{2r + 5}{r + 2}$ is a better approximation to $\sqrt{5}$.
3. Let P be a polygon whose vertices have integer coordinates and whose sides have integer lengths. Prove that the perimeter of P is an even number.
4. Two distinct numbers a and b are chosen at random from the set $\{2, 2^2, 2^3, \dots, 2^{25}\}$. What is the probability that $\log_b(a)$ is an integer?
5. In the picture below, the largest circle has radius 1, and there are infinitely many smaller circles packed into the corner, each as large as possible. What is the area of the shaded region?

