

### SLU Math Team 2008 Qualifying Problems

Do as much as you can, and return your work to Dr. Clair on or before  
Monday, April 7.

1. Four spheres of radius 1 are stacked so that each is tangent (externally) to the other three. What is the radius of the largest sphere that can fit into the space between them?
2. The Missouri Lotto drawing fills a bin with balls numbered 1-49, then draws five balls, one at a time without replacement. What is the probability that the balls are drawn in increasing numerical order?
3. Determine whether the improper integral

$$\int_0^{\infty} (-1)^{\lfloor x^2 \rfloor} dx$$

converges or diverges, where  $\lfloor \cdot \rfloor$  is the greatest integer function.

4. Let  $f$  and  $g$  be functions from the set  $\mathbb{R}$  of real numbers to itself, such that  $g(x) < f(x)$  for all  $x \in \mathbb{R}$ . Prove there exists an infinite subset  $S \subseteq \mathbb{R}$  such that  $g(x) < f(y)$  for all  $x, y \in S$ .
5. If  $f$  is a polynomial of degree  $n$  such that  $f(i) = 2^i$  for  $i = 0, 1, \dots, n$ , find  $f(n+1)$ .