SLU Math Team 2007 Qualifying Problems

Do as much as you can, and return your work to Dr. Clair on or before Tuesday, March 20.

1. Prove that

$$\sqrt{3+\sqrt{3+\sqrt{3+\cdots}}}$$

exists and find its exact value.

2. SLU's box at Busch stadium has six seats arranged in two rows of three. If Father Biondi and five rich alumni go to a game, how many ways can the six people sit so that each person in the front row is shorter than the person directly behind them in the back row. Assume all six people are different heights.

3. Let $\alpha \geq 1$ and let $f(x) = x^\alpha$. Let $b(x)$ be the y-intercept of the normal line to f at $(x, f(x))$. What is $\lim_{x\to 0+} b(x)$?

4. Let $B_1 = 0$ and $B_2 = 1$. For $n > 2$, the number B_n is defined by writing the decimal digits of B_{n-1} followed by the digits of B_{n-2}. For example $B_3 = B_2B_1 = 10$, $B_4 = B_3B_2 = 101$ and $B_5 = B_4B_3 = 10110$. Determine all n so that 11 divides B_n.

5. Let T be the tetrahedron with vertices $(0,0,0)$, $(a,0,0)$, $(0,b,0)$, and $(0,0,c)$ for some $a,b,c > 0$. Let V be the volume of T and let ℓ be the sum of the lengths of the six edges of T. Prove that

$$V \leq \frac{\ell^3}{6(3+3\sqrt{2})^3}$$