2006 SLU Math Team Qualifying Problems – Solutions:

1. Can win with 170 using T20, T20, DB. Cannot win with 159.

2. Call the function f. Suppose f has more than one root in [-1,1]. Then there is a point c in (-1,1) with $f'(c) = 0$ by Rolle's Theorem. However, $f'(x) = 3x^2 - 3$ has roots only at +1 and -1.

3. Put $k = i + j$, so $j!$ becomes $(k - i)!$ Apply the binomial theorem to eliminate one summation. The result is e^{x+1}. It's also possible to organize the terms as a power series with coefficients that depend on i. The limit of the nth coefficient is $e/n!$, so the series sums to ee^x. (Purdue Math Problem of the Week, #9 Fall 2002)

4. The pythagorean theorem says $(DE)^2 + (DF)^2 = (EF)^2$. This implies the areas of the semicircles based on DE and DF add to the area of the semicircle based on EF. The areas of the moons plus the area of the semicircle based on EF equal the area of the triangle plus the areas of the semicirlces based on DE and DF (a picture would help). These two observations prove the result.

5. Canadian Math Olympiad, 1999 problem #3.