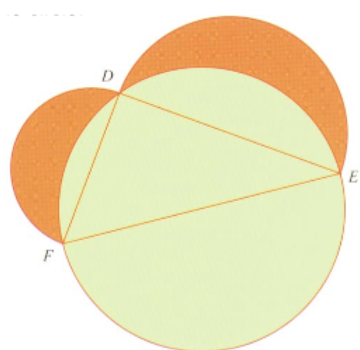
2006 SLU Math Team Qualifying Problems
Due Tuesday, March 21

1. In professional darts, players begin with 501 points and take turns throwing "rounds" of three darts. The points scored by those three darts are subtracted from their score, and the first player to reach zero wins. The dartboard has regions worth 1,2,3,...,20 points, and each of these numbered regions has a subregion worth double and another worth triple points (e.g. "double 13" is worth 26 points). In addition, there is a bullseye region worth 25 and a double bullseye region worth 50 (there is no triple bullseye). A dart can miss the board and score 0. In the final round that reduces a player's score to zero, the third dart is required to land in a double zone.
What is the highest score that a player can have and still win with one round?
What is the lowest score that a player can have, yet be unable to win with one round?

2. Prove that $x^3 - 3x + b$ cannot have more than one zero in [-1,1], regardless of the value of b.

3. Determine $\lim_{n \to \infty} \sum_{i=0}^{n} \sum_{j=0}^{n-i} \frac{x^j}{i!\,j!}$

4. Given right triangle DEF, construct circles with centers on the midpoints of the three sides, as shown below. Show that the sum of the areas of the two shaded moons is equal to the area of triangle DEF.



5. For any positive integer n, let $d(n)$ be the number of positive divisors of n. Find all n with the property that $n = (d(n))^2$.