

Solutions:

1. Instead, do 0000-9999 with all leading zeros. Then $1/10$ of the digits are zero, and there are 40000 digits written, so 4000 zeros. There are $1000 + 100 + 10 + 1$ extra zeros written, so 2889 zeros from 1-9999 and four extra makes 2893.
2. The two sides are equal at $x=1$, and their difference has derivative 0 only at $x=1$, and is concave up. (Bjorn Poonen's inequalities handout)
3. There are 24 half-faces, 24 with one edge a long-diagonal, and 8 equilateral (side $\sqrt{2}$). Hence $12 + 12\sqrt{2} + 4\sqrt{3}$. (American Invitational Mathematics Examination 1, 2003)
4. The diameter of the circle is $\sqrt{2}$. Let P be a corner of the small square and on the circle. Draw OP. Extend the edge of the small square containing P towards the center, getting closest at Q. The right triangle OPQ and the Pythagorean theorem finish the problem. (AIME 2, 1998?)
5. Take the last four digits $abcd$. Since addition of either 0 or 1 on the right destroys the property, both subsequences $abcd0$ and $abcd1$. Thus the four digits $abcd$ occur three times. If none of them is positioned in the beginning, then each of them is a subsequence of either $0abcd$ or $1abcd$, which is impossible. (New Zealand Mathematical Olympiad Camp, 1998)