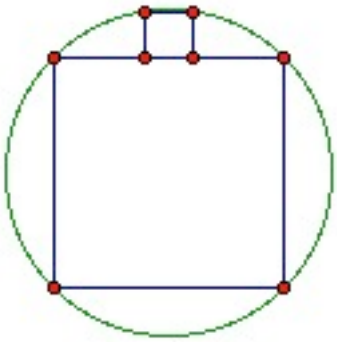
2005 SLU Math Team Qualifying Problems
Due Tuesday, March 15

1. Write the numbers from 1 to 10000. How many zeros did you write?

2. Prove that $2\sqrt{x} \geq 3 - \frac{1}{x}$ for all $x > 0$.

3. Let S be the set of vertices of a unit cube. Find the sum of the areas of all triangles whose vertices are in S.

4. If the large square has area 4, find the area of the small square.



5. Suppose a finite sequence $b_1, b_2, \ldots, b_n$ of 0's and 1's has the property

   (*) For any $i \neq j$, the subsequence $b_i, b_{i+1}, b_{i+2}, b_{i+3}, b_{i+4}$ is different from the subsequence $b_j, b_{j+1}, b_{j+2}, b_{j+3}, b_{j+4}$.

   and suppose the sequences $b_1, b_2, \ldots, b_n, 0$ and $b_1, b_2, \ldots, b_n, 1$ do not have the property (*).
   Prove that the first four digits of the sequence $b_1, b_2, \ldots, b_n$ are the same as its last four digits.