

Problem 1 Solution

The black area is 4 times bigger than the white one.

Let the length of the card be "a"

so, the biggest(first) black area is $a \cdot a/2 + a^2/4$ (IT'S $3/4(a^2)$)

the next black area is $(a/2) \cdot (a/4) + a^2/16$ (IT'S $3/16(a^2)$)

So, the sum of black area is $3 \cdot a^2(1/4 + 1/64 + \dots + 1/(4^n))$ (n is a positive odd number(n=1,3,5,...))

The same, we can see, the sum of white area is $3 \cdot a^2(1/16 + \dots + 1/(4^m))$ (m is a positive even number(m=2,4,6,...))

So, the ratio of the total area of the black squares to total area of the white squares is $(1/4 + 1/64 + \dots + 1/(4^n)) / (1/16 + \dots + 1/(4^m)) (3 \cdot a^2$ cancel out)

$$\begin{aligned}
&= (4^2 + 4^4 + \dots + 4^m) / (4^1 + 4^3 + \dots + 4^n) \\
&= 4(4^1 + 4^3 + \dots + 4^n) / (4^1 + 4^3 + \dots + 4^n) \\
&= 4
\end{aligned}$$

Problem 2 Solution

<http://mathforum.org/dmpow/solutions/solution.ehtml?puzzle=57>Solution:
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They can sit in 336 different ways.

Bonus: My formula is:

$$[(n - 2x + 2) * 1 + (n - 2x + 1) * 2 + (n - 2x) * 3 + \dots + 1 * (n - 2x + 2)] * (x_P_x)$$

I used the following notation:

- A the first customer
- B the second customer
- C the third customer.

I made the following algorithm:
the order doesn't have importance.
there are 10 places at the counter

Because there is one seat between them, every person will receive two seats, So if there are three customers at the counter the third can stay on one of the following seats (6):

+ + + + + + + + + +
A - B - C C C C C C

If B moves on the next seat, C can stay on 5 seats.
...

If A moves on the next seat there are $5 + 4 + 3 + 2 + 1$ possibilities.

If A moves on the next seat there are
 $4 + 3 + 2 + 1$ possibilities.

...

The number of possibilities is:

$$\begin{array}{r} 6 + 5 + 4 + 3 + 2 + 1 + \\ 5 + 4 + 3 + 2 + 1 + \\ 4 + 3 + 2 + 1 + \\ 3 + 2 + 1 + \\ 2 + 1 + \\ 1 + \\ \hline 6 * 1 + 5 * 2 + 4 * 3 + 3 * 4 + 2 * 5 + 1 * 6 = \\ 6 * 2 + 10 * 2 + 12 * 2 = \\ 12 + 20 + 24 = \\ 56 \text{ (possibilities)} \end{array}$$

But the customers can be arranged in any order. So:

$$56 * \frac{3!}{0!} = 56 * \frac{3 * 2 * 1}{1} = 56 * 6 = 336$$

there are 336 possibilities.

Bonus

I changed the values into letters and I obtained the following generalisation:

$$[(n - 2x + 2) * 1 + (n - 2x + 1) * 2 + (n - 2x) * 3 + \dots + 1 * (n - 2x + 2)] * (x_P_x)$$

The expression $(n - 2x + 2)$ is obtained in the following way:

- there are n places
- I'm interested to know how many possibilities are for the last customer, when the first is placed on the first place, the second at a place difference (third position), and so on, up to the $(x - 1)$ -th. So, I must keep $2 * (x-1)$ places for these $(x-1)$ persons, and there are:

$$n - 2 * (x-1) = n - 2x + 2$$

places left for the last one.

Now, I verify for the data in the problem: $n = 10$, $x = 3$.

$$n - 2x + 2 = 6.$$

The right paranthesis could be written:

$$6 * 1 + 5 * 2 + 4 * 3 + 3 * 4 + 2 * 5 + 1 * 6,$$

this means the same as in the solution presented before.