# SLU Mathematics Team Qualifier
2004

Please write solutions to problems on separate sheets of paper, with your name on each sheet. Justify or prove all your answers.

1. In the picture below, assume the alternating bands of black and white continue forever toward the upper left corner. At each step, the relevant line segments are exactly half the size from the previous step. What is the ratio of white area to black area?
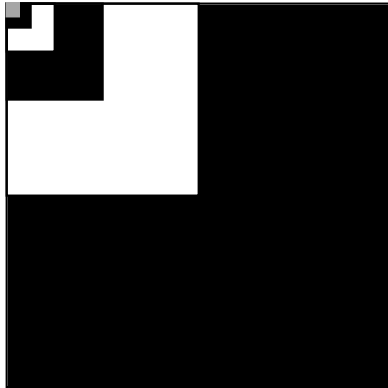
2. Alice, Bob, and Carol don't like each other. At a bar with a 10 stool counter, how many ways can they sit so that no two sit on neighboring stools?

3. Let $p(x)$ be the parabola $x^2 + ax + b$. The two rays $y = x$ $(x > 0)$ and $y = 2x$ $(x > 0)$ define wedge shaped area, and suppose that a and b are chosen so that two disjoint arcs of the parabola cross through the wedge. Show that the projection of the left-hand arc onto the x-axis is smaller than the projection of the right-hand arc onto the x-axis by exactly 1.

4. Place one positive integer at each vertex of a cube, so that all your numbers are different. On each edge of the cube, write the greatest common divisor of the numbers at its two endpoints. Can the sum of the edge numbers equal the sum of the vertex numbers?