

## Homework 5

Due Monday, September 28

WMMY: Ch 4 # 33, 35, 45

Problem A: Suppose  $X$  has the uniform distribution on the interval  $[a,b]$ . That is, the PDF for  $X$  is  $f(x) = 1/(b-a)$  when  $x \in [a,b]$ , and  $f(x) = 0$  otherwise.

- Find  $\mu(X)$  and  $\sigma(X)$ .
- Sketch the PDF for  $X$  and display the mean  $\mu$  and the range of one standard deviation about the mean (from  $\mu-\sigma$  to  $\mu+\sigma$ ).

Problem B: Suppose a website gets 100 hits per second. A reasonable model of the time  $T$  between successive hits has PDF  $f(t) = 100e^{-100t}$  for  $t > 0$ .

- Find  $\mu(T)$  and  $\sigma(T)$ .
- Sketch the PDF for  $T$  and display the mean  $\mu$  and the range of one standard deviation about the mean (from  $\mu-\sigma$  to  $\mu+\sigma$ ).

Problem C: Prove Theorem 4.2 in the case when  $X$  is a continuous random variable.

Problem D: Let  $X$  and  $Y$  have the joint PDF which is uniform on the unit disk. That is,  $f(x,y) = 1/\pi$  if  $x^2+y^2 \leq 1$  and  $f(x,y) = 0$  otherwise.

Find  $E(X)$ ,  $E(Y)$ , and the correlation coefficient  $\rho_{XY}$ . This is an example where  $X$  and  $Y$  have correlation 0 but are not independent.

Problem E: Let  $X$  and  $Y$  have the joint PDF which is uniform on the unit disk, as in Problem C. Suppose you repeatedly pick points in the disk using this distribution. On average, how far do you expect to be from the center of the disk?

More precisely, this problem is asking you to compute the expected value of  $\sqrt{X^2+Y^2}$ .

Hint: This is not hard, but you need to change to polar coordinates.