

## Reading

- Joseph, pages 461 (bottom) - 487
- Stillwell, Chapter 6 (except we're not covering 6.2 and 6.6)
- MacTutor History of Mathematics archive, *Tartaglia versus Cardan*

## Exercises

### Lattice Multiplication

Compute  $1990 \times 365$  using lattice multiplication.

Compute  $(x^2 + 3x - 1)(2x^2 - x + 5)$  using lattice multiplication.

**Quadratic Equation** Page 478 of Joseph has a diagram of al-Khwarizimi's geometric solution to the quadratic  $x^2 + 10x = 39$ . Make up another example and draw the corresponding diagram.

### Fibonacci Numbers

Assuming  $\lim_{n \rightarrow \infty} \frac{F_{n+1}}{F_n}$  exists, prove it is the golden ratio  $\frac{1+\sqrt{5}}{2}$ .

### Polynomial Equations

1. Solve  $x^3 + 60x^2 + 1200x = 4000$ . Hint: complete the cube. (Dardi of Pisa, 1344)
2. Antonio de Mazzinghi (1353-1383): Find two numbers such that multiplying one by the other makes eight and the sum of their squares is twenty-seven.  
Solve this in two ways: 1) By eliminating one variable and solving the resulting fourth degree equation with the quadratic equation. 2) using de Mazzinghi's technique of setting the two numbers to  $a + \sqrt{b}$  and  $a - \sqrt{b}$ .  
Check numerically that your two answers are the same.
3. Express  $\sqrt{27 + \sqrt{200}}$  as  $a + \sqrt{b}$  for integer  $a$  and  $b$ .  
(From *Coss*, by Christoph Rudolff. This 1525 book introduced the notation  $\sqrt{\quad}$  for roots.)

**Cubic Equations** Give exact answers to all of these, then find a decimal approximation.

1. Solve  $x^3 + 24x = 16$ .
2. Solve  $x^3 + 27x = 6x^2 + 58$ .
3. Solve the problem that Zuanne da Coi proposed to Tartaglia in 1530: Find a number which multiplied by its root increased by three equals five.
4. Fior's first question to Tartaglia in the challenge of 1535: Find the number when added to its cube root gives six.
5. Tartaglia's second question to Fior in the challenge of 1535: Find an irrational quantity such that when it is multiplied by its square root diminished by thirty, the result is a given rational number.
6. Write one (or more) scathing insults you would cast upon a mathematician who cannot solve cubic equations. Bonus points for period language.