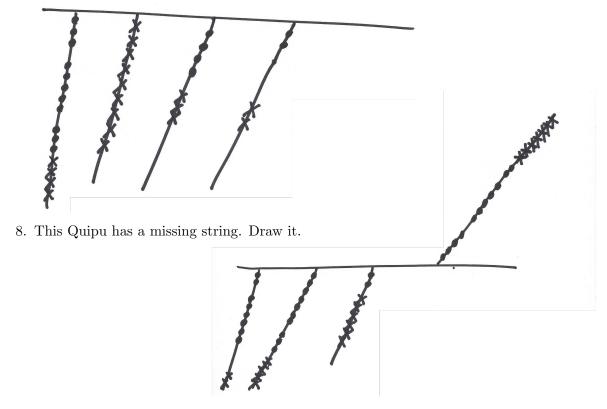## Reading

- Stillwell, Chapter 3.4, 5.4, 5.5, 5.7

- Joseph, pages 372-392

- There are some handouts on our webpage, and also sections in Joseph (see the presentation assignment) that you may need to understand the number systems section of this homework.

Exercises

Number Systems (Class Presentations)

1. Translate from Yoruban to a modern Arabic numeral: *meji din ogota*

2. Translate 72 to a Yoruban spoken number.

3. Write out how to get these numbers using the Yoruba base 20 system:
   (a) 143
   (b) 65
   (c) 99

4. Write 1,000,000 in Mayan numerals.

5. Convert $\left|\begin{array}{c} :|| \\ \cdot| \end{array}\right|$ and $\left|\begin{array}{c} \vdots \\ \cdot|| \\ \oplus \end{array}\right|$ to modern Arabic numbers.

6. Draw a Quipu string that signifies 7359 using • for short knots and × for long knots.

7. Draw the missing top string for this Quipu:

8. This Quipu has a missing string. Draw it.

9. Write out, using Chinese zongs and hengs, $396 + 122 = 518$.

10. Solve this division problem with the Chinese numeral method: 2120426/281.

11. Solve $1524 \div 127$ using the Egyptian method, in hieroglyphics.

12. Determine the best way to split three loaves of bread amongst seven people. Here, "best" means the way the Egyptians would have preferred, so that one person is not stuck with many small leftover pieces. (Hint: use a sum of unit fractions).

Pell's Equation

1. Find a positive integer solution (x, y) for:

   (a) $x^2 - 7y^2 = 1$

   (b) $x^2 - 22y^2 = 1$

   (c) $x^2 - 56y^2 = 1$

   (d) (optional - challenge!) $x^2 - 58y^2 = 1$

2. Find four positive integer solutions to $x^2 - 2y^2 = -1$.

3. Find three positive (even) integers n so that $n + 1$ and $\frac{n}{2} + 1$ are both squares.

4. Find four Pythagorean triples (a, b, c) where a and b are consecutive integers.

   Hint: $p^2 - q^2 - 2pq = (p - q)^2 - 2q^2$.

   Remark: These give rational right triangles that closely approximate the $45° - 45° - 90°$ triangle with sides $(1, 1, \sqrt{2})$.