MATH 4050 Homework 3 Due Wednesday, February 10 (or whenever)

Reading

- Stillwell, Chapter 3, 5.1, 5.3-5.5, 5.7 (note we aren't using 3.4, 3.5, 5.4, 5.5 yet)
- Joseph, pages 380-392

Exercises

Formulas For Primes

1. (Euler's polynomial, 1772) Is it true that $p(x) = x^2 + x + 41$ is prime for every $x = 0, 1, 2, \ldots$?
2. Show that the Mersenne number $M_n = 2^n - 1$ is composite when n is composite, so that the only possible Mersenne primes are M_p for p prime.
   (Hint: Write $n = ab$ and $M_n = 2^{ab} - 1 = (2^a)^b - 1^b$)

GCD and Linear Diophantine Equations

1. Stillwell: # 5.3.1, 5.3.2
2. For each of these, decide if it can be solved. If so, find any solution:
   (a) $24x + 138y = 18$
   (b) $14x + 35y = 87$
   (c) $221x + 35y = 11$
3. Suppose that x_0, y_0 is a solution to the Diophantine equation $ax + by = c$. Assume that $\gcd(a, b) = 1$, and show that all solutions to the equation are given by $x = x_0 + bt$, $y = y_0 - at$ for integer values of t.
4. (Alcuin of York, 775AD) A hundred bushels of grain are distributed among 100 persons in such a way that each man receives 3 bushels, each woman 2 bushels, and each child $\frac{1}{2}$ bushel. How many men, women, and children are there?
5. (Mahaviracarya, 850AD) There were 63 equal piles of plantain fruit and 7 single fruits. They were divided evenly among 23 travelers. What is the number of fruits in each pile?