

Reading

- Stillwell, 21.8, 22.9, 24.7*, 24.8*, 24.9
(* these are much too hard, but have some choice tidbits - do the best you can)
- Finish *Logicomix*.
- You may find D. Royster's chapters on hyperbolic geometry helpful

Exercises

Hyperbolic Geometry

1. A *Saccheri quadrilateral* is a quadrilateral $ABCD$ where angles C and D are right angles, and sides AD and BC have the same length. Using absolute geometry, prove that angles A and B are equal.
2. Prove the 'angle-angle-angle' theorem for hyperbolic geometry: Two triangles whose corresponding angles are equal are congruent. Hint: line them up at one corner, and if they are not the same triangle the space between them is a quadrilateral with a problem.
3. For each part, draw a Poincaré disk and then draw the indicated diagram.
 - (a) Four hyperbolic lines that don't cross each other
 - (b) A point P , a hyperbolic line ℓ , and three more hyperbolic lines through P that don't cross ℓ .
 - (c) A triangle with angles $90^\circ - 5^\circ - 5^\circ$.
 - (d) A Saccheri quadrilateral with acute angles of 45° .
 - (e) A pentagon with five right angles.
4. Find the defect of each hyperbolic polygon in question 3c, d, and e.

Hilbert's Problems

Choose one of Hilbert's 23 problems and give a short explanation of it (or part of it), and to what extent it has been solved. Some of the more approachable ones are 1, 2, 3, 7, 8, 10, 13, 16, 17 and 18.

Russell's Paradox

Here is a slight variation on Russell's paradox. Let S be any set (which contains sets). Let R be the set of $x \in S$ with $x \notin x$. That is, R is all sets in S which do not contain themselves. Modern set theory axioms allow R to exist.

- (a) Show that R is not in S .
- (b) Explain why there can be no set containing all sets.
(Cantor is very excited about this, on pg. 169 of *Logicomix*).