Due Tuesday Feb 9.
Written portion due in class.

CS 3820 Assignment 3

Read Angel Chapter 4.3, 4.7-4.10

Rotation Matrices

Let $\mathbf{R}_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$. For example, $\mathbf{R}_{30^\circ} = \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix}$

(a) What is $\mathbf{R}_{45^\circ}$?

(b) Let $\vec{u} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$, $\vec{v} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$, and $\vec{w} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$. Sketch these three vectors.

(c) Sketch the vectors $\mathbf{R}_{45^\circ}\vec{u}$, $\mathbf{R}_{45^\circ}\vec{v}$, $\mathbf{R}_{45^\circ}\vec{w}$.

(d) What effect did multiplication by $\mathbf{R}_{45^\circ}$ have on the three vectors?

(e) What effect does multiplication by $\mathbf{R}_{120^\circ}$ have on the three vectors?

(f) Explain why $\mathbf{R}_\theta$ is called a"rotation matrix".

(g) Show that $\mathbf{R}_\alpha\mathbf{R}_\beta = \mathbf{R}_{\alpha+\beta}$. (Hint: use a sum-of-angles identity.)

(h) What does the equation of part (g) tell you about rotations?

Commutativity

Let $\mathbf{S} = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ and $\mathbf{T} = \begin{pmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{pmatrix}$. Let $P = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$ be a point.

(a) Geometrically, what does multiplication by $\mathbf{S}$ do to P?

(b) Geometrically, what does multiplication by $\mathbf{T}$ do to P?

(c) Geometrically, what does multiplication by $\mathbf{TS}$ do to P?

(d) Geometrically, what does multiplication by $\mathbf{ST}$ do to P?