

## Discrete Math – Take Home Quiz 2

This quiz should take you approximately 25 minutes. There are 5 questions, worth a total of 50 points. You may use reference material, but are not allowed to ask for help from anyone except Dr. Clair.

- (10) 1. Suppose  $n$ ,  $r$ , and  $k$  are integers and  $1 \leq k \leq r \leq n$ . Prove that

$$\binom{n}{r} \binom{r}{k} = \binom{n}{k} \binom{n-k}{n-r}$$

**Solution:**

$$\binom{n}{r} \binom{r}{k} = \frac{n!}{r!(n-r)!} \cdot \frac{r!}{k!(r-k)!} = \frac{n!}{k!(r-k)!(n-r)!}$$

$$\binom{n}{k} \binom{n-k}{n-r} = \frac{n!}{k!(n-k)!} \cdot \frac{(n-k)!}{(n-r)!(r-k)!} = \frac{n!}{k!(r-k)!(n-r)!}$$

You can also make a combinatorial argument. Both sides choose three sets  $A \supset B \supset C$ , where  $|A| = n$ ,  $|B| = r$ , and  $|C| = k$ . The left hand side takes  $A$ , picks  $r$  of these to form  $B$ , and then picks  $k$  out of  $B$  to form  $C$ . The right hand side takes  $A$ , picks  $k$  of them to form  $C$  and some of  $B$ , and then picks  $n-r$  of  $A-C$  to fill out the rest of  $B$ .

- (10) 2. Have you ever ordered a burrito at Chipotle? First, they want to know your filling: steak, carnitas, chicken, chorizo, barbacoa, sofritas, or veggie. Then, you pick brown or white rice, then black or pinto beans. You pick one of four types of salsa. Finally, you may add any of: lettuce, sour cream, cheese, and/or guacamole.

How many different Chipotle burrito orders are possible?

**Solution:**  $7 \cdot 2 \cdot 2 \cdot 4 \cdot 2^4 = 1792$ .

(10) 3. (a) What is the coefficient of  $x^{12}y^3$  in the polynomial  $(x + y)^{15}$ ?

(b) What is the coefficient of  $x^{12}$  in the polynomial  $(x + 10)^{15}$ ?

**Solution:** a.  $\binom{15}{12} = 455$ ; b.  $10^3 \cdot \binom{15}{12} = 455000$ .

(10) 4. Select five cards at random from a standard deck of 52 cards. What is the probability that your hand contains exactly two Aces?

**Solution:**  $\frac{\binom{4}{2} \cdot \binom{48}{3}}{\binom{52}{5}} \approx 0.03993$ , about 4%.

(10) 5. (a) How many six letter strings use only the letters A, E, R, S, T?

(b) How many of these have no double letters (the same letter twice in a row)?

(c) (One bonus point) Find 10 English words that fit part (b).

**Solution:** a.  $5^6 = 15625$ ; b.  $5 \cdot 4^5 = 5120$ ; c. aerate asters eaters eraser erases estate  
esters rarest raster raters resets retest serest starer stares starts stater states strata  
tartar tarter taster tastes taters teaser teases terser tester testes tetras treats tsetse