Due Monday, Feb. 13 at start of class   Name: ____________________

Discrete Math – Take Home Quiz 1

This quiz should take you approximately 25 minutes. You may use reference material, but are not allowed to ask for help from anyone except Dr. Clair.

(10) 1. Show that $(q \vee s) \wedge (r \vee p) \wedge (\neg s \vee \neg p) \wedge (s \vee \neg q)$ is satisfiable.

Solution: Let $p = F$, $r = T$, $s = T$ (and q can be either T or F).

(10) 2. Suppose x and y are integers. True or false:

(a) __**T**__ $\forall x(x^2 \geq x)$

(b) __**T**__ $\exists x(x^2 \leq x)$

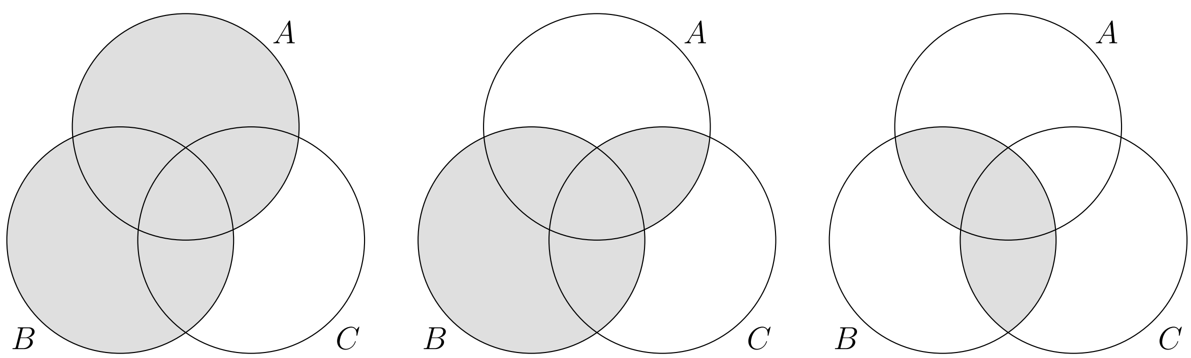
(c) __**F**__ $\exists! x(x^2 = 9)$

(d) __**F**__ $\forall x(x^2 > 0 \rightarrow x > 0)$

(e) __**F**__ $\forall x \exists y(x = y^2)$

(10) 3. In the Venn diagrams below, A, B, and C are the three circular regions.

Describe each shaded set in terms of A, B, C and basic set operations.



Solution: $A \cup B$ $B \cup (A \cap C)$ $B \cap (A \cup C)$

(there are other ways to answer correctly)

(10) 4. Suppose a, b, c are positive real numbers.

Prove that if $abc > 1000$ then one of a, b, or c is larger than 10.

Solution: I will prove the contrapositive. Suppose $a \le 10$, $b \le 10$, and $c \le 10$. Multiplying these together gives $abc \le 1000$. So, if $abc > 1000$ then one of a, b, or c is larger than 10.

(10) 5. Suppose $x, y \in \mathbb{Z}$ with $x^2 = 1 + 13y^2$. Show that exactly one of x and y must be odd.

Extra credit: Find an example of x and y.

Solution: Split into two cases if y is odd or even.

If y is odd, then y^2 is odd, $13y^2$ is odd, and $1 + 13y^2 = x^2$ is even. So x is even.

If y is even, then y^2 is even, $13y^2$ is even, and $1 + 13y^2 = x^2$ is odd. So x is odd.

In both cases exactly one of x and y are odd.

Note that I used facts about sums and products of even and odd numbers that we have already proved.

One solution is $x = 1$, $y = 0$. I meant to rule this out, but it *is* a solution, so you got some extra credit for finding it. The smallest interesting solution is $x = 649, y = 180$. If you are curious about this, look up "Pell's Equation".