Probability (with cards)

A standard deck of cards has 4 suits (spades, hearts, diamonds, clubs) and in each suit there are 13 ranks (2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A), making $4 \times 13 = 52$ cards total.

Blackjack

In blackjack, there is a player and a dealer. The dealer gets two cards, one face up and one face down. The player also gets two cards. The cards have values given by their rank, with J, Q, K all worth 10, and A worth 1 or 11 at the player's choice.

1. The best hand is *blackjack*, which means you get an Ace and a value ten card (10, J, Q, K). What is the probability of getting blackjack?

   Solution: $\dfrac{4 \cdot 16}{\binom{52}{2}} \approx 4.83\%$

2. If you get two cards of the same rank, you can *split*. What is the probability you can split?

   Solution: $\dfrac{13 \cdot \binom{4}{2}}{\binom{52}{2}} \approx 5.88\%$

3. If your hand is a 16, you have two cards that add to 16. A *hard* 16 has no Ace. The worst blackjack hand is a hard 16 which cannot be split. What is the probability you get it?

   Solution: $\dfrac{16 \cdot 4 + 4 \cdot 4}{\binom{52}{2}} \approx 6.03\%$

Poker

In five card poker, you are dealt five cards.

4. What is the probability you are dealt *four of a kind*: all four cards of one rank, and one other card?

   Solution: $\dfrac{13 \cdot 48}{\binom{52}{5}} \approx 0.024\%$

5. What is the probability you are dealt a *full house*: three cards of one rank, and two of another?

$$\text{Solution: } \frac{13 \cdot 12 \cdot \binom{4}{3} \cdot \binom{4}{2}}{\binom{52}{5}} \approx 0.144\%$$

6. What is the probability you are dealt a *flush*: all five cards are the same suit?

$$\text{Solution: } \frac{4 \cdot \binom{13}{5}}{\binom{52}{5}} \approx 0.198\%$$

7. What is the probability you are dealt a *pair*: two cards of one rank, and three other cards.

$$\text{Solution: } \frac{13 \cdot \binom{4}{2} \cdot \binom{12}{3} \cdot 4^3}{\binom{52}{5}} \approx 42.3\%$$

War
In the card game "war", there are two players and each gets half the deck (26 cards).

8. What is the probability that you are dealt all four aces?

$$\text{Solution: } \frac{\binom{48}{22}}{\binom{52}{26}} \approx 5.52\%$$

9. What is the probability that you win the game?

Solution: $1/2$

Bridge (or Hearts, Spades, other trick taking games)
In these games, there are four players and each player gets 13 cards.

10. What is the probability you get all 13 cards of the same suit?

> **Solution:** $\dfrac{4}{\binom{52}{13}} \approx 6.3 \times 10^{-12}$

11. What is the probability you get a *flat hand*, which means four of one suit and three of each of the others?

> **Solution:** $\dfrac{4 \cdot \binom{13}{4} \cdot \binom{13}{3}^3}{\binom{52}{13}} \approx 10.5\%$

12. What is the probability you get a seven card suit?

> **Solution:** $\dfrac{4 \cdot \binom{13}{7} \cdot \binom{39}{6}}{\binom{52}{13}} \approx 3.53\%$

Answers:
1. 4.83%; 2. 5.88%; 3. 6.03%; 4. 0.024%; 5. 0.144%; 6. 0.198%; 7. 42.3%;
8. 5.52%; 9. 50%; 10. 6.3×10^{-12}; 11. 10.5%; 12. 3.53%