Math 1660

Big O

1. Give a big-O estimate for each of these functions.
   (a) $\dfrac{x^3}{1000} + 1000x^2 + 1000000$
   (b) $\left(n^3 + n\log(\log(n))\right)(n\log(n) + 2n + \log(n))$
   (c) $\left(x + \dfrac{1}{x}\right)\log(x) + x + \sqrt{x+1}$
   (d) $\sqrt{x^4 + \log(x^9)}$
   (e) $2^n + n^3 + n^2 + n\log(n)$
   (f) $\dfrac{n(n+1)(2n+1)}{6}$
2. Let c be a constant, and suppose f is $O(g)$. Prove that cf is $O(g)$.
3. Show that 2^n is $O(3^n)$ but 3^n is not $O(2^n)$. (This means $2^n \ll 3^n$.)
4. Suppose a computer can perform 10^{20} divisions per second (this is wildly optimistic). How long will it take to check all factors of N up to $\sqrt{N}$, where N is a 100-digit integer?