Math 1660

Proof practice for Exam 1

1. Prove $C \subseteq A \cap B \to C \subseteq A \wedge C \subseteq B$.

    Solution: Let $x \in C$. Since $C \subseteq A \cap B$, $x \in A \cap B$, and so $x \in A$ and $x \in B$.
    Since $x \in A$, $C \subseteq A$. Since $x \in B$, $C \subseteq B$.
    So $C \subseteq A \wedge C \subseteq B$.

2. Prove $A \cup B \subseteq C \to A \subseteq C \wedge B \subseteq C$.

    Solution:
    Let $x \in A$. Then $x \in A \cup B$. Since $A \cup B \subseteq C$, $x \in C$, and this shows $A \subseteq C$.
    Let $x \in B$. Then $x \in A \cup B$. Since $A \cup B \subseteq C$, $x \in C$, and this shows $B \subseteq C$.

3. Prove $C \subseteq A - B \to B \cap C = \emptyset$

    Solution: I will prove this by contradiction. Suppose $B \cap C \neq \emptyset$. Then there is some $x \in B \cap C$. So $x \in B$, and $x \in C$. Since $C \subseteq A - B$, $x \in A - B$. This means $x \in A \wedge x \notin B$. So $x \notin B$. But earlier, I showed $x \in B$. This is a contradiction, so $B \cap C = \emptyset$.

4. Prove $\emptyset \subset A$.

    Solution: For all x, $x \notin \emptyset$. So $x \in \emptyset$ is always false. Then $x \in \emptyset \to x \in A$ is true (vacuously). So $\emptyset \subset A$.

5. Show, for $n \in \mathbb{Z}$, that if $2|n$ and $3|n$ then $6|n$.

    Solution: Suppose $2|n$ and $3|n$. Then there are $h, k \in \mathbb{Z}$ with $n = 2h$ and $n = 3k$. So $2h = 3k$. Since $2h$ is even, $3k$ must be even, so k must be even. Then $k = 2\ell$ for some ℓ. Then $n = 3k = 3(2\ell) = 6\ell$ and this shows $6|n$.

6. Show that if x and y are rational, then $x + y$ is rational.

Solution: Since x is rational, there are integers p and q with $x = \frac{p}{q}$.
Since y is rational, there are integers r and s with $y = \frac{r}{s}$.
Then

$$x + y = \frac{p}{q} + \frac{r}{s} = \frac{sp + qr}{qs}$$

so $x + y$ is rational.