Math 1660

Proof Practice

1/30/17

Prove directly

1. If n is an even integer, then a^2 is even.
2. Let T be a right triangle with legs a, b, and hypotenuse c. If $c = \sqrt{2ab}$ then T is isosceles.
3. If n is an even integer and $n > 2$, then $2^n - 1$ is composite (not prime).

Prove the contrapositive

4. If x^3 is irrational then x is irrational.
5. If $a + b > 0$ then $a > 0$ or $b > 0$.
6. Suppose n, a, and b are integers, and n factors as $n = ab$. Then one of a and b must be less than or equal to $\sqrt{n}$.

Prove by contradiction

7. $\sqrt[3]{2}$ is irrational.
8. Let T be a right triangle with legs a, b, and hypotenuse c, and suppose a, b, and c are all integers. Then a, b, and c cannot all be odd.
9. If there are 10 people at a party, two of them have the same number of friends at the party.

Prove

10. If p is a prime number, then $p + 7$ is composite.
11. If xy is irrational, then x is irrational or y is irrational.
12. If a and b are consecutive integers, then $(a + b)^2$ is odd.
13. Suppose $x > 0$ and $y > 0$. Then $\frac{x+y}{2} \geq \sqrt{xy}$.
    (This is the arithmetic geometric mean inequality.)