Week 9 Exercises

- (Boothby pg. 205 #1) For V a vector space with $\dim V > 1$, show $\mathcal{T}^r(V) = \bigwedge^r(V) \oplus \sum^r(V)$ when $r = 2$ but not for $r > 2$.
- (Boothby pg. 214 #6) Definition and basic properties of interior product. The hard part is the product rule. One way to prove this is 'by force', which Lee does in Proposition 8.13. You should prove it in the special cases when $r = 1$ and $s = 1, 2, 3$. Then, either prove (or accept) that it's true for $r = 1$ and all $s > 1$ - it should be clear at this point, and only the notation makes it hard. Then you can use induction on r, proving that if the product rule holds for r and all s, then it holds for $r + 1$ and all s.

  Extending to tensor fields on a manifold is trivial. Just replace v's with X's. It's not worth re-writing everything.
- (Lee Ch 8 # 3) A criterion for functions to be coordinate charts. You previously did something like this with Lee's Chapter 2 #17.
- Give a natural geometric criterion for a basis $\mathbf{u}, \mathbf{v}$ of $\mathbb{R}^2$ to be positively oriented.
- For which n is $\mathbb{R}P^n$ orientable?