Week 14 Exercises

1. For $n \in \mathbb{Z}$, let P_n be the 0-manifold consisting of n points $p_1, \ldots, p_n$. Suppose $f : P_n \to P_m$ is any map (obviously smooth).
   (a) Using the natural basis for H^0 describe the matrix $f^* : H^0(P_m) \to H^0(P_n)$.
   (b) When is f^* injective? When is f^* surjective?

2. Suppose M is a manifold with finite fundamental group. Show that $H^1(M) = 0$.
   Hint: The universal cover $\tilde{M}$ of M is simply connected, and $\pi_1(M)$ is a finite group of diffeomorphisms of $\tilde{M}$.

3. Let $M = S^1$ with coordinate θ, and let t be the coordinate on $\mathbb{R}$. Define a (not particularly special) one-form on $M \times \mathbb{R}$ by:

$$\omega = t \cos\theta dt + t^2 \sin\theta d\theta.$$

   With K and s_a as in Lee, Theorem 10.8, check that $(id - \pi^* s_a^*)\omega = (dK - Kd)\omega$.

4. Define the cup product $\cup : H^k(M) \times H^\ell(M) \to H^{k+\ell}(M)$ by

$$[\alpha] \cup [\beta] = [\alpha \wedge \beta].$$

   Show $\cup$ is well defined and bilinear.