Week 8-9 Exercises

1. Let X be the sterographic coordinate on S^1, given by $(x, y) \to \frac{x}{1-y} = X$. Write $\frac{\partial}{\partial X}$ in terms of the vector fields $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}$ on $\mathbb{R}^2$, and show $\frac{\partial}{\partial X}$ extends to a vector field on all of S^1.

2. Define a free and proper action of $\mathbb{Z}$ on $\mathbb{R}^3$ by $1 \cdot (x, y, z) = (x + 1, -y, -z)$. The quotient E has a vector bundle structure over S^1 with projection $\pi : E \to S^1$ by $\pi(x, y, z) = e^{2\pi i x}$. Show that E is a trivial $\mathbb{R}^2$ bundle over S^1, hence diffeomorphic to $S^1 \times \mathbb{R}^2$.

3. Read over section 2.4, which shows how $T_{(p,q)}(M \times N) = T_pM \times T_pN$. Show that if M and N are parallelizable manifolds, then so is $M \times N$.