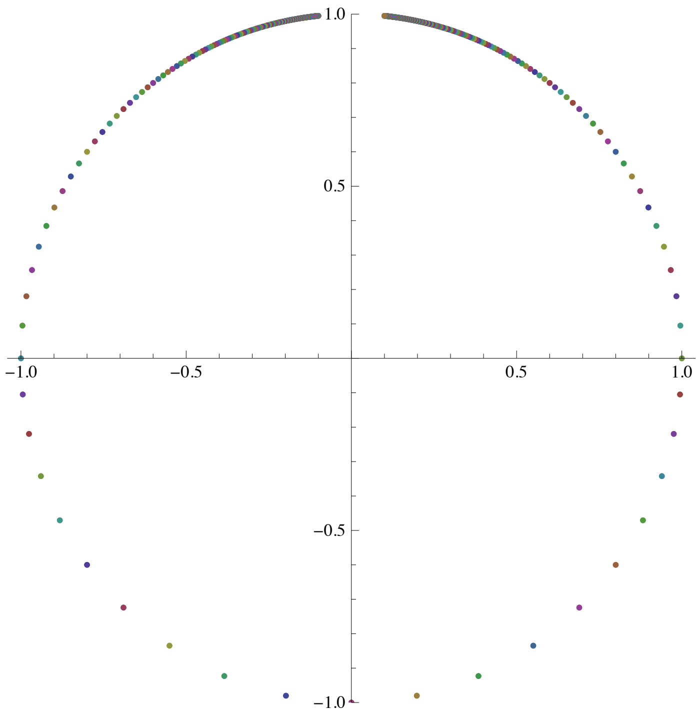
Solution for Week 8-9 # 1

Inverse stereographic map:

```
f[X_] := {2 X, X^2 - 1} / (1 + X^2)
```

```
ListPlot[Table[{f[X]}, {X, -20, 20, .1}], PlotRange → {-1, 1}, AspectRatio → 1]
```



The Jacobian of f:

```
Jf = D[f[X], X]
```

$$\left\{-\frac{4\,X^2}{\left(1+X^2\right)^2}+\frac{2}{1+X^2},\ -\frac{2\,X\left(-1+X^2\right)}{\left(1+X^2\right)^2}+\frac{2\,X}{1+X^2}\right\}$$

Change to (x,y) coordinates and simplify on the unit circle:

```
Simplify[Jf /. {X → x / (1 - y)}, Assumptions → (x^2 + y^2 == 1)]
```

$\{(-1+y)\,y,\ x-x\,y\}$

The vector field $\partial/\partial X$ in (x,y) coordinates:

```
v[x_, y_] := {y (y - 1), x (1 - y)}
```

Here’s how it looks, tangent everywhere to the unit circle:

```
Show[Graphics[Table[
   Arrow[{{Cos[t], Sin[t]}, {Cos[t], Sin[t]} + v[Cos[t], Sin[t]]}], {t, -Pi, Pi, Pi / 20}]]]
```