Week 3 Exercises

1. Lee, Exercise 1.17: Show that a manifold is connected if and only if it is path connected

2. Show that Brouwer's Invariance of Domain implies that a connected manifold has well defined dimension.

3. Lee, Exercise 1.48: The manifold $\mathbb{R}P^1$ is one-dimensional. Give a diffeomorphism between it and the circle.

4. Show that the quotient of the two-sphere by the antipodal map $\mathbf{x} \to -\mathbf{x}$ is diffeomorphic to $\mathbb{R}P^2$. Show that $\mathbb{R}P^2$ is diffeomorphic to a square with opposite edges identified via reversing orientations:

5. Let M be the set of triangles in $\mathbb{R}^2$. Give an atlas on M to show it is a manifold.