Week 13 Exercises

1. Prove Lee Proposition 2.76 (the Jacobi identity and two other basic facts about Lie bracket)

2. Given a unit vector $\mathbf{u}$ in $\mathbb{R}^3$, let $X_{\mathbf{u}}$ be the clockwise rotation field for $\mathbf{u}$.

   Compute $[X_{\mathbf{u}}, X_{\mathbf{v}}]$ for unit vectors $\mathbf{u}$,$\mathbf{v}$.

   For $\mathbf{x} \in S^2 \subset \mathbb{R}^2$, the field is given by the cross product $X_{\mathbf{u}}(\mathbf{x}) = \mathbf{u} \times \mathbf{x}$. Note that in the usual spherical parameterization $(\theta, \varphi) \to (\cos(\theta)\cos(\phi), \sin(\theta)\cos(\phi), \sin(\phi))$, the field $X_{\mathbf{k}} = \frac{\partial}{\partial\theta}$.

3. Let $\sigma(t) = r\cos(t)\mathbf{i} + r\sin(t)\mathbf{j} + t\mathbf{k}$ be a helix with radius r. Find the value of r which maximizes the curvature of σ.

4. Let $\sigma(t) = e^t\cos(t)\mathbf{i} + e^t\sin(t)\mathbf{j}$ for t any real number. Reparameterize σ by the arclength parameter s, and compute the length of σ for t from $-\infty$ to 0. This is a curve which spirals infinitely many times around the origin but has only finite length.