Math 641

Week 12 Exercises

1. Lee Exercise 3.10: Steiner's Roman surface as an immersion of $\mathbb{R}P^2$ into $\mathbb{R}^3$ and an embedding into $\mathbb{R}^4$. Beware that in the definitions of f and g, Lee is assuming $(x, y, z) \in S^2$, i.e. $x^2 + y^2 + z^2 = 1$.

2. When do $\mathbb{R}^k \times \{0\}$ and $\{0\} \times \mathbb{R}^\ell$ intersect transversally in $\mathbb{R}^n$?

3. Let V be a vector space, and let Δ be the diagonal of $V \times V$. For a linear map $A : V \to V$, let $\Gamma = \{(v, Av) | v \in V\}$ be the graph of A. Show that Γ intersects Δ transversally if and only if 1 is not an eigenvalue of A.

4. Suppose M_1 and M_2 are regular submanifolds of N which intersect transversally. Show $M_1 \cap M_2$ is a submanifold of N. What is its dimension?

5. Lee Exercise 2.29: Transversality of composed maps. There is a typo in the problem statement, which should read: $f \pitchfork g^{-1}(W)$ if and only if $(g \circ f) \pitchfork W$.