Week 11 Exercises

1. Lee Exercise 3.6: Show that every injective immersion of a compact manifold is an embedding.

2. Show that any two loops in $\mathbb{R}^3$ can be nudged away from each other. More precisely:

   Given smooth maps $f : S^1 \to \mathbb{R}^3$ and $g : S^1 \to \mathbb{R}^3$, show that for any $\delta > 0$ there is a vector $\mathbf{v} \in \mathbb{R}^3$ with $||\mathbf{v}|| < \delta$ so that the sets $\{f(x)\}_{x \in S^1}$ and $\{g(x) + \mathbf{v}\}_{x \in S^1}$ are disjoint.

3. Let M be any (paracompact) manifold which is not second countable. For example, M could be an uncountable disjoint union of circles. Show there is no embedding $M \to \mathbb{R}^n$ for any n, so the Whitney embedding theorem fails.